Assignment #9

Due *Monday 28 April, 2014.*

Read sections 4.1 through 4.6 of Morton & Mayers, 2nd ed.

1. Exercise 4.1 in Morton & Mayers (page 146).

2. Reproduce Figure 4.6 on page 99 of Morton & Mayers, 2nd ed. The equation being solved, the initial condition, and the exact solution are all stated on page 98. The spatial interval is $[0, 1]$.

3. (*This is a simplified version of Exercise 4.4 in* Morton & Mayers, *on page 147.*) Suppose a is a positive constant. Determine the coefficients c_{-1}, c_0, c_1, so that the scheme

$$U_j^{n+1} = c_{-1}U_{j-1}^n + c_0 U_j^n + c_1 U_{j+1}^n$$

for the solution of the equation $u_t + au_x = 0$ agrees with the Taylor series expansion of $u(x_j, t_{n+1})$ to as high an order as possible. In particular, find three equations in the three unknowns c_{-1}, c_0, c_1 so that

$$u(x, t + \Delta t) \qquad \text{and} \qquad c_{-1}u(x - \Delta x, t) + c_0 u(x, t) + c_1 u(x + \Delta x, t)$$

agree to as high order as possible, when one expands $u(x-\Delta x, t)$, $u(x+\Delta x, t)$, $u(x, t+\Delta t)$ in Taylor series around the basepoint (x, t). You will use the fact that u solves $u_t+au_x = 0$ at (x, t), but, unlike the derivation on page 101, you will *not* use any finite difference approximations to derivatives.

Verify that the result is the Lax-Wendroff scheme given by (4.36) on page 100 of Morton & Mayers. (*In equation (4.36) and on pages 100-101, note that* $\nu = a\Delta t/\Delta x$*.*)

4. The code

`bueler.github.io/M615S14/ftcs.m`

was used in class to demonstrate that the forward-time, centered-space (FTCS) method was unstable even for very short time steps. Modify this code, renaming it `laxwendroff.m`, to implement the Lax-Wendroff scheme on the *same problem* as solved by `ftcs.m`. The boundary conditions can be implemented the same way. Use the CFL condition to determine the time step. Confirm that it works by computing the worst case error on the whole mesh,

$$\max_{j,n} |U_j^n - u(x_j, t_n)|$$

and showing that this decreases as Δx (and thus Δt, by the CFL) decreases. Concretely, look at $J = 50, 100, 200, 400$ grids, plus finer grids if you want, and show a convergence plot with error-versus-Δx.