Assignment #5

Due *Monday, 10 March, 2014* at start of class.

See the online slides: `http://bueler.github.io/M615S14/twopoint.pdf`

1. Solve by-hand this ODE BVP to find $y(x)$:

$$y'' + 2y' + 2y = 0, \qquad y(0) = 1, \quad y(1) = 0.$$

2. Recall Example 3 in the slides, an impossible-to-solve ODE BVP. Nonetheless there are some values of A in the following problem which allow a solution: find $y(x)$ if

$$y'' + \pi^2 y = 0, \qquad y(0) = 1, \quad y(1) = A.$$

What values of A are allowed? For an allowed value of A, how many solutions are there?

3. Apply the finite difference method to solve this ODE BVP:

$$y'' + \sin(5x)y = x^3 - x, \qquad y(0) = 0, \quad y(1) = 0.$$

In particular, use $J = 10$, $\Delta x = 1/J$, and $x_j = j\Delta x$ for $j = 0, \dots, J$. Construct the system

$$A\,\mathbf{y} = \mathbf{b}$$

where A is a $(J + 1) \times (J + 1)$ matrix, $\mathbf{y} = \{Y_j\}$ approximates the unknowns $\{y(x_j)\}$, and $\mathbf{b}$ contains the right-side function "$x^3 - x$" in the ODE. Arrange things so that the first equation in the system represents the boundary condition "$y(0) = 0$" and the last equation the condition "$y(1) = 0$". The remaining equations in the system will each hold finite difference approximations of the ODE. Solve the system to find $\mathbf{y}$, and plot it appropriately. Write a couple of sentences addressing how to know qualitatively and quantitatively whether your answer is a good approximation.

4. (*The goal of this problem is to understand shooting. You will not quite put all parts together, however. With the knowledge from this problem you could make a program like* `varheatSHOOT.m`, *which uses bisection to converge to an A value so that $u(1) \approx 0$ to many-digit-accuracy.*)

Consider the nonlinear ODE BVP

$$u'' + u^3 = 0, \qquad u(0) = 1, \quad u(1) = -2.$$

This problem is well-suited to the shooting method. Specifically, write a Matlab program that uses an ODE solver to solve the following ODE IVP

$$u'' + u^3 = 0, \qquad u(0) = 1, \quad u'(0) = A$$

for each of the eleven values $A = -5, -4, \dots, 4, 5$. Plot all eleven solutions, and identify on the plot[1] the A value for each curve. Which two A values make the computed value $u(1)$ bracket the desired boundary condition value "$u(1) = -2$"?

[1] Use the `text` command in Matlab.