Assignment #2

Due *Friday, 7 February 2014.*

Read sections 2.2, 2.3, 2.4, 2.5 and 2.6 of Morton & Mayers, 2nd ed.

1. Suppose that f and its derivatives f', f'', and f''' are continuous on an interval around x. Suppose also that there is $M \ge 0$ so that $|f'''(y)| \le M$ for all y in this interval. Use $n = 2$ Taylor's theorem with remainder to compute $f(x+2h)$ and $f(x+h)$. From these, show carefully that

$$\left| f'(x) - \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h} \right| \le 2Mh^2.$$

This inequality tells us the accuracy of the "one-sided" finite difference approximation

$$f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}.$$

Compare what we know about this one-sided method to what we know about the centered formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

which has been considered in lecture.

2. Write a Matlab program which applies explicit scheme (2.19), in Morton & Mayers on page 11, to solve the following heat equation problem:

$$\begin{aligned} u_t &= K u_{xx} \quad \text{for } t > 0, \quad 0 < x < L, \\ u(0,t) &= A \quad \text{for } t > 0, \\ u(L,t) &= B \quad \text{for } t > 0, \\ u(x,0) &= G(x) \quad \text{for } 0 < x < L. \end{aligned}$$

Unlike (2.7)–(2.9) it has nonzero (nonhomogeneous) Dirichlet boundary conditions A and B, a coefficient $K > 0$, and an x-interval of arbitrary length L. Note I would recommend that you change (2.20) to say

$$\mu = \frac{K\Delta t}{(\Delta x)^2}.$$

In particular, write a Matlab function which is called like this:

```
expscheme(J,L,N,tf,A,B,K,G)
```

where `J` is the number of x-subintervals, `L` is the length of the x-interval, `N` is the number of time-steps, `tf` is the final time, and `A,B,K,G` are used in the heat equation problem above. Turn in your code. To show it works, also turn in the plot of the solution at the final time $u(x,t_f)$, from the command

```
expscheme(20,3,100,2,1,-4,0.1,@(x) exp(-x))
```

3. Do exercise 2.1 in Morton & Mayers (pages 56–57).

4. Reproduce Figure 2.2 in Morton & Mayers (page 13) using Matlab. Print your resulting reproduction and turn in the program(s) that make it.

> *Commentary.* Figure 2.2 shows solutions of problem (2.7)–(2.9) using initial condition (2.24). It includes both the exact solution from the Fourier series (2.11) and approximate solutions from the explicit scheme (2.19).
>
> My suggestion when reproducing Figure 2.2 is to first write a program that will plot a truncation of the exact solution (2.11). (This requires you know the coefficients a_m, but see exercise 2.1.) Then write a different program to compute the explicit approximation from scheme (2.19); see problem 2 above. This second program will step forward using Δt indicated in Figure 2.2; you can fix $\Delta x = 0.05$. Then display both the exact and approximate solutions as in Figure 2.2, possibly in a third program which calls the first two programs.
>
> Try to reproduce all the essential features of Figure 2.2. Use `subplot` and also `hold`. Use the plot appearance option as in `plot(x,y,'.-')`; see `help plot`.