Assignment #1

Due *Monday, 27 January 2014.*

Read *lightly* the introduction (Chapter 1) of the textbook Morton & Mayers.
Read *seriously* sections 2.1, 2.2, 2.3, and 2.4 of the textbook.

1. (*There is nothing to turn in on this problem.*) Find textbooks on *calculus* and *ordinary differential equations* (ODEs). You will need these references throughout the semester. Review these two topics, the first of which may be best explained by an additional *numerical analysis* textbook:

i) Taylor's theorem with remainder formula, and
ii) the solution of linear homogeneous constant-coefficient ODEs.

2. Calculate $(81.2)^{1/4}$ to within $0.00001 = 10^{-5}$ without any computing machinery except a pencil. Prove that your answer has this accuracy. (*Hint: You may use a computer to* check *your by-hand value.*)

3. Assume f' is continuous. Derive the remainder formula

$$\int_0^a f(x)\,dx = a f(0) + \frac{1}{2} a^2 f'(\nu) \tag{1}$$

for some (unknown) ν between zero and a. (*Hint:* $f(x) = f(0) + f'(\xi)x$ *where* $\xi = \xi(x)$ *is some number between* 0 *and* x.) Use two sentences to explain the meaning of (1), as an answer to the question "how accurate is the left-hand endpoint integration rule $\int_0^a f(x)\,dx \approx a f(0)$?"

4. Solve, by hand,

$$y'' + 5y' + 4y = 0, \quad y(-1) = 2, \quad y'(-1) = 0, \tag{2}$$

for the solution $y(t)$, and then find $y(3)$. On t, y axes, show the initial values, the solution, and the value $y(3)$. Note this is a *prediction* of the outcome at $t = 3$, given initial data at $t = -1$ and a "law", namely the differential equation itself, about how $y(t)$ varies in time.

5. Download and/or install and/or find Matlab (or Octave or pylab). Now work at the command line to compute a finite sum approximation to

$$\sum_{n=1}^{\infty} \frac{1}{n^3}.$$

Compute at least three partial (finite) sums, with increasing numbers of terms N. Do you think you are getting close to the infinite sum? Finally, turn your command line work into a function `sumthirdpower(N)`, and show that it works. Turn in both the command line session (be brief) and the code `sumthirdpower.m`.

6. Using Euler's method for approximately solving ODEs, write your own Matlab program (either script or function) to solve initial value problem (2) to find $y(3)$. Use a few step sizes, decreasing as needed, so that you get apparent four digit accuracy. (*Hint: You can use a built-in ODE solver to* check *your work, but this is not required.*)

7. (*This Fourier series problem is a warmup for exercise 2.1 in the textbook* Morton & Mayers, *an exercise which will appear on the next assignment. I* encourage *you to see wikipedia pages and other resources about Fourier series!*)

(a) Assume n and m are integers, with $n \geq 1$ and $m \geq 1$. Show that

$$\int_0^1 \sin(n\pi x)\sin(m\pi x)\,dx = \begin{cases} 0, & n \neq m, \\ \frac{1}{2}, & n = m. \end{cases}$$

(b) Assume that a_n (for $n = 1, 2, \dots$) are real numbers ("coefficients") and assume that

$$f(x) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)$$

defines a function on $[0, 1]$. Show that

$$a_m = 2\int_0^1 f(x)\sin(m\pi x)\,dx.$$

(c) Find the coefficients a_n so that

$$x = \sum_{n=1}^{\infty} a_n \sin(n\pi x)$$

on the interval $0 \leq x \leq 1$. Use Matlab to plot the $N = 1, 2, 3, 5, 20, 50$ partial sums

$$s_N(x) = \sum_{n=1}^{N} a_n \sin(n\pi x)$$

on $[0, 1]$, also showing $f(x) = x$ in your plot.

(d) Show that

$$\left|\sum_{n=1}^{N} a_n \sin(n\pi x)\right| \leq \sum_{n=1}^{N} |a_n|.$$

For the coefficients in part **(c)**, does

$$\sum_{n=1}^{\infty} |a_n|$$

converge?