Assignment #7

Due *Monday April 9, 2012.*

Read sections 3.1, 3.2, 4.1, and 4.2 of MORTON & MAYERS.
Note that Version 1.0 of your project is due Friday April 6.

1. Reproduce Figures 3.2, 3.3, 3.4, and 3.5 from MORTON & MAYERS. Even though they appear in section 3.2, they were produced by the explicit scheme in 3.1, so all I am asking you to implement is the explicit scheme; of course you need to figure out and use a stable timestep! Use only a $J_x = 100$ by $J_y = 100$ grid; don't worry about finer grids. Your solution will be greatly assisted by using this code I have already written to compute the initial condition:

`http://www.dms.uaf.edu/~bueler/formM.m`

2. ***(a)*** For the problem

$$u_t + (2 - t^2)u_x = 0, \qquad u(x, 0) = \arctan x,$$

and for $x \in \mathbb{R}$ and $t \geq 0$, sketch the characteristics in the (x, t) plane. In particular, sketch at least six characteristics which appear in the rectangle $-3 \leq x \leq 3$ and $0 \leq t \leq 4$. (*Hints for a good sketch*: What are the initial slopes? At what time does the advection change direction?)
(b) Solve the problem by hand. That is, apply the method of characteristics. Check your answer $u(x, t)$ by substitution into the PDE. Your answer should be correct for $t \geq 0$ and any x.
(c) Write a MOP program to just plot the exact solution $u = u(x, t)$. In particular, plot this solution as a mesh on the rectangle $-3 \leq x \leq 3$, $0 \leq t \leq 4$, and as a contour plot on the same rectangle. (*The contours will be familiar curves ...*)

3. Exercise 4.1 in MORTON & MAYERS, 2ND ED (page 146).