

Assignment #2

Due *Monday, 6 February 2012.*

MOP = MATLAB, OCTAVE, or PYLAB

Read sections 2.5 and 2.6 of MORTON & MAYERS, 2ND ED. This Assignment relates especially to sections 2.1 through 2.5.

1. Suppose that f is continuous, and also that the derivatives f' , f'' , and f''' are continuous on an interval around x . Suppose also that there is $M \geq 0$ so that $|f'''(y)| \leq M$ for all y in this interval. Using Taylor's theorem with remainder to compute $f(x+2h)$ and $f(x+h)$, show carefully that

$$\left| f'(x) - \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h} \right| \leq 2Mh^2.$$

This inequality tells us the accuracy of the “lop-sided” finite difference approximation

$$f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}.$$

Compare what we know about this lop-sided method to what we know about the centered formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

which has been carefully considered in lecture.

2. Do exercise 2.1 in MORTON & MAYERS (pages 56–57).

3. Reproduce Figure 2.2 in MORTON & MAYERS (page 13) using MOP.

(*This figure shows solutions of problem (2.7)–(2.9) using initial condition (2.24). It includes both the exact solution from the Fourier series (2.11) and approximate solutions from the explicit scheme (2.19). A suggested procedure is to first write a program that will plot a truncation of the exact solution (2.11), with the correct coefficients a_m . Then write a different program to compute the explicit approximation. This second program will step forward using (2.19); you can fix $\Delta x = 0.05$ but your values from Δt are from Figure 2.2. Then display both the exact and approximate solutions as in the Figure, possibly in a third program, or at the command line. Plot using MOP reasonably carefully to reproduce all the essential features of Figure 2.2. In MATLAB/OCTAVE use `subplot` and also `hold`. Use the plot appearance option as in `plot(x,y,'.-')`. At the end it would be fine to combine everything into one clear program, or you can clearly present the parts.*)