Assignment #1

Due *Monday, 30 January 2012.*

Read *lightly* the introduction of the textbook MORTON & MAYERS. Read *seriously* subsections 2.1, 2.2, 2.3, and 2.4 of the textbook.

1. (*There is nothing to turn in on this problem.*) Find textbooks on *calculus* and on *ordinary differential equations* (ODEs). Generally, you will need these references to recall ideas which arise in Math 615.

Review these two topics, the first of which may be best explained by a *numerical analysis* textbook:

i) Taylor's theorem with remainder formula, and
ii) the solution of linear homogeneous constant-coefficient ODEs.

2. Calculate $(27.2)^{1/3}$ to four correct (total) digits without any computing machinery except a pen(cil). (*Hint: Please use a calculator to* check *your by-hand value.*) Prove that your answer is within 0.0005 of the correct answer, again without any computing machinery. (*Hint: Use one of the topics in problem* **1** *above.*)

3. This problem is about the question "How accurate is this left-hand endpoint integration rule $\int_0^a f(x)\,dx \approx a f(0)$?" Assume f' is continuous. Derive the remainder formula

$$\int_0^a f(x)\,dx = a f(0) + \frac{1}{2}a^2 f'(\nu) \tag{1}$$

for some (unknown) ν between zero and a. (*Hint:* $f(x) = f(0) + f'(\xi)x$ *where* $\xi = \xi(x)$ *is some number between* 0 *and* x.) Use two sentences to explain the meaning of (1) to the layperson.

4. Solve, by hand,

$$y'' + 5y' + 4y = 0, \quad y(1) = 0, \quad y'(1) = 2, \tag{2}$$

for the solution $y(t)$, and then find $y(3)$. On t, y axes, show the initial values, the solution, and the value $y(3)$. Note this is a *prediction* of the outcome at $t = 3$, given initial data at $t = 1$ and a "law" about how $y(t)$ varies in time.

5. *This is a write-your-first-*MOP*-program kind of problem.*

Download and/or install and/or find MOP = (MATLAB, OCTAVE, or PYLAB). Now work at the command line to compute a finite sum approximation to

$$\sum_{n=0}^{\infty} \frac{1}{n^4}.$$

Compute at least three partial (finite) sums, with increasing numbers of terms N. Do you think you are getting close to the infinite sum? Finally, turn your command line work into a saved function `sumfourthpower(N)`, and show that it works.

6. Using Euler's method for approximately solving ODEs, write your own MOP program (either script or function) to solve initial value problem (2) to find $y(3)$. Use a few step sizes, decreasing as needed, so that any reasonable observer would agree that you have four digit accuracy. (*Hint: Learn how to use a built-in ODE solver to check your work; not required.*)