Assignment #5

Due Wednesday 14 October, 2015 at the start of class

There will be no class on Friday 9 October, 2015. Please read slides at

bueler.github.io/M614F15/iterative.pdf

P10. Use MATLAB to compute the 2-norm condition numbers for systems LS1 and LS2 in the slides. (*Thereby confirm that these systems have unique solutions which can be well-approximated.*) Find the exact solutions of these systems. (*For example, use* MATLAB *any way you want, and then check that solution by-hand.*)

P11. Write a MATLAB function for Richardson iteration, with first line

```
function z = richardson(A,b,x0,omega,N)
```

It should return the Nth iterate $\mathbf{x}_N$ as `z`. Confirm that it works by showing you get the same $\mathbf{x}_3$ as on page 4 of the slides. What is a preferred value for ω in system LS1? How many iterations are needed to get 8 digit accuracy for LS1 with $\mathbf{x}_0 = 0$ and this preferred value of ω?

P12. Find a small example matrix A which has all zeros on the diagonal but which is invertible. Find its inverse.

P13. Write *two* MATLAB functions for Gauss-Seidel iteration with first lines

```
function z = gs1(A,b,x0,N)
function z = gs2(A,b,x0,N)
```

For `gs1()`, implement formula (7) from the slides by carefully using MATLAB functions `triu()` and `tril()` to extract the parts of A, and then using backslash. For `gs2()` implement (8) by using only scalar arithmetic operations, and `for` loops. (I.e. pretend it is old Fortran.)

Demonstrate that the two versions work identically on LS1 by computing two iterations with each. How many iterations are needed to get 8 digit accuracy for LS1 using $\mathbf{x}_0 = 0$? After demonstrating that Gauss-Seidel iteration fails on LS2, compute a spectral radius that explains why it fails.

P14. Show that Jacobi iteration converges if A is strictly diagonally-dominant. (*Hints: Jacobi iteration converges if and only if* $\rho(M) < 1$ *for* $M = -D^{-1}(L + U)$*. So suppose* $M\mathbf{v} = \lambda\mathbf{v}$ *for* $\mathbf{v} \neq 0$*. Choose the largest-magnitude entry* v_i *of* $\mathbf{v}$*, so that* $|v_i| \geq |v_j|$ *for all* j*. Show then that* $M\mathbf{v} = \lambda\mathbf{v}$*, and the assumption of strict diagonal dominance, shows* $|\lambda v_i| < |v_i|$ *which shows* $|\lambda| < 1$*.*)