Assignment #9

Due *Tuesday, 2 May 2006.*

I. Read the handout "The hydrogen atom and the periodic table," lecture III.19 from *The Feynman Lectures in Physics*. Note the sketch Fig. 19-6, which I will not attempt to reproduce.

II. Again, read chapter 19 from Riley, Hobson, & Bence, paying special attention to expansions in "spherical polars." Read chapter 24 very lightly.

III. Do exercises on the rotation group in the plane:

Exercise P. Let G be the set of all 2×2 matrices of the form

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

for some $\theta \in [0, 2\pi)$. Show that G satisfies the definition of a *group* on page 885, where the product operation "$\bullet$" is ordinary matrix multiplication in this case. Give a formula for R_θ^{-1}. [*Regarding the associative law, but only that law, you may* refer *to a clear reference if you find one. Note that the standard name for this G is "$SO(2)$," a name which I will explain in class.*]

Exercise Q. Do the following exercise *by hand*; the point is that you must think through where points go. I don't care if you are a bad sketch artist—I want you to think about the pictures associated to the action of rotations on vectors and on functions!

(a) Let $\theta_1 = \pi/6$, $\theta_2 = 3\pi/2$. Pick two vectors $\mathbf{v}_1, \mathbf{v}_2$ in the plane, with different directions and different nonzero lengths just to make the picture fairly general, and draw a clear picture showing all of the following six vectors in the same diagram:

$$\mathbf{v}_1, \mathbf{v}_2, R_{\theta_1}\mathbf{v}_1, R_{\theta_1}\mathbf{v}_2, R_{\theta_1}\mathbf{v}_1, R_{\theta_1}\mathbf{v}_2.$$

(b) Given

$$f(x, y) = f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right),$$

an ordinary scalar function on the plane with its input point thought of as a vector, and given an element R_θ of the rotation group $G = SO(2)$ above, define

$$(R_\theta f)(x, y) = f\left(R_\theta^{-1}\begin{bmatrix} x \\ y \end{bmatrix}\right)$$

for all points (x, y) in the plane. For each of θ_i in part **(a)**, and for the two functions $g(x, y) = x^2$ and $h(x, y) = (x - y)^{-2}$, sketch the graphs of $z = g(x, y)$, $z = h(x, y)$, $z = (R_{\theta_1} g)(x, y)$, $z = (R_{\theta_1} h)(x, y)$, $z = (R_{\theta_2} g)(x, y)$, $z = (R_{\theta_2} h)(x, y)$ in six separate figures. [*Yes, I am asking for six different 3D surface graphs.*] Finally, explain in a couple of sentences why "R_θ^{-1}" occurs in the definition of the action of R_θ on a function f.

IV. Do one big exercise from Riley, Hobson, & Bence:

19.17.

[*Note that in the interior of the spherical shell the potential u solves $\nabla^2 u = 0$. The hint on this problem says "you will need to use the result of exercise 17.7," which is true.*]