Math 612 Mathematical Physics II (Bueler)

Assignment #8

Due *Tuesday, 25 April 2006.*

I. Read chapter 19 from Riley, Hobson, & Bence, paying special attention to expansions in "cylindrical polars" and "spherical polars," that is, to what an American would call "cylindrical" and "spherical" coordinates. Read chapter 24 very lightly.

II. Do exercises from Riley, Hobson, & Bence on *separation of variables*:

19.4, 19.11, 19.13.

III. Do exercises on *spherical (harmonic) expansions*, that is, on separation of variables of spherically-symmetric systems in spherical coordinates:

Exercise M. [*Replacement of Exercise 19.5 in* Riley, Hobson, & Bence.] For each of the following two functions in spherical coordinates, (i) show that $\nabla^2 u = 0$, and (ii) identify indices l, m and coefficients A, B, C, D, E, F so that the given function is in the form (19.49).

$$\text{(a)} \qquad u(r,\theta,\phi) = \left(5r^2 + \frac{7}{r^3}\right) \frac{3\cos^2\theta - 1}{2}$$

$$\text{(b)} \qquad u(r,\theta,\phi) = \left(11r + \frac{13}{r^2}\right) \sin\theta \, \exp i\phi$$

[*Note that the answer for indices is unique but that for coefficients it is not. Note that in part* (**b**) *the strict answer is that "one cannot write in the form of (19.49)," but that one can write the answer in a series, each term of which is in the form of (19.49).*]

Exercise N. [*Replacement of Exercise 19.7 in* Riley, Hobson, & Bence.] Verify that for $l = 0, 1, 2$ the following identity is true:

$$\sum_{m=-l}^{l} |Y_l^m(\theta,\phi)|^2 = \frac{2l+1}{4\pi}.$$

Use the formulas on page 671 for $Y_0^0, Y_1^0, Y_1^{\pm 1}, Y_2^0, Y_2^{\pm 1}, Y_2^{\pm 2}$.