Math 612 Mathematical Physics II (Bueler)

Assignment #4

Due *Tuesday, 28 February 2006.*

I. Read sections 20.10 through 20.20; sections 20.11 and 20.12 apply to this assignment.

II. Do exercises:

Exercise H. Show that if C is a positively-oriented simple closed curve then the area A of the region enclosed by C can be written

$$A = \frac{1}{2i} \oint_C z^* \, dz.$$

[*Hint*: Though the integrand is not analytic, Green's theorem does apply; look it up or use the divergence theorem to get to it as I did in class. Note that A is real!]

Exercise I. Let C be any simple closed curve in the z plane, traversed in the positive sense, and define

$$g(w) = \int_C \frac{z^3 + 2z}{(z-w)^3} \, dz.$$

Show that $g(w) = 6\pi i w$ when w is inside C and that $g(w) = 0$ when w is outside C.

Exercise J. Show that if f is analytic in a domain R containing a (fixed) simple closed curve C and if z_0 is in R but not on C then

$$\int_C \frac{f'(z)\, dz}{z - z_0} = \int_C \frac{f(z)\, dz}{(z - z_0)^2}.$$

Exercise K. Let C be the unit circle $z = e^{i\theta}$, $-\pi \le \theta \le \pi$. Show that if $a \in \mathbb{R}$ then

$$\int_C \frac{e^{az}}{z} \, dz = 2\pi i.$$

Show that, as a consequence,

$$\int_0^\pi e^{a\cos\theta} \cos(a \sin\theta) \, d\theta = \pi. \tag{1}$$

[*For 2 points extra credit, verify this numerically for a couple of values of a. For 2 more points extra credit, do integral* (1) *without complex analysis, using only calculus.*]

Exercise L. Do the integral problem printed on the back of this sheet.

III. Do exercises from Riley, Hobson, & Bence:

20.13.