Assignment #3

Due *Tuesday, 14 February 2006.*

I. Read the "*86. Linear fractional transformations*" excerpt from Churchill & Brown. See also the "*Appendix 2. Table of transformations of regions*" excerpt. These supplement the rather sparse treatment of conformal maps in sections 20.8 and 20.9.

II. Do exercises:

Exercise E. Find a linear fraction transformation that maps the points $z_1 = \infty$, $z_2 = i$, $z_3 = 0$ to the points $w_1 = 0$, $w_2 = i$, $w_3 = \infty$.

Exercise F. Find a linear fraction transformation that maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ to the points $w_1 = 1$, $w_2 = i$, $w_3 = -1$. What is the image of the line $y = x$ (in the input z space) under this transformation?

Exercise G. (a) Show that the composition of two linear fractional transformations is a linear fractional transformation.
(b) Let $T(z) = (az + b)/(cz + d)$ where $ad - bc \neq 0$. Show that $T^{-1} = T$ if and only if *either* $T(z) = z$ *or* $d = -a$. [*Hint*: Write the equation $T^{-1}(z) = T(z)$ as $(a + d)\left[cz^2 + (d - a)z - b\right] = 0$.]

III. Do exercises from Riley, Hobson, & Bence:

20.9, 20.12