Final Exam (Take-Home)

Due *Friday, 16 December, 2005* at 5:00pm in my office mailbox.

Each problem is worth 10 points for a total of 110 points.

Rules. *Please do not consult with any human being on this exam other than me, Ed Bueler. You may refer to existing text and electronic material, but any significant use must be referenced; this includes significant results from the textbook. You may* check *your answers through use of technology, but only your by-hand work will be graded. All arguments must be complete. I will assume that if major steps are missing then you* do not *know how to fill them in.*

1. Exercise 8.7.

2. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & \alpha \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{pmatrix},$$

where α, β are *nonzero* complex numbers.
(a). Find the eigenvalues and eigenvectors of A.
(b). Give conditions on α, β so that all of the eigenvalues are real.
(c). Give conditions on α, β so that a spanning set of *orthogonal* eigenvectors exists.
(d). Show that the conditions in *(b)* and *(c)* are simultaneously satisfied if and only if A is Hermitian. [*Hint*: You need to show that if *(b),(c)* both apply then Hermitian, *and* vice versa. Also, to show that *(b),(c)* imply Hermitian, write α, β in polar form.]

3. Exercise 8.15.

4. Let

$$M = \begin{pmatrix} 9 & -4 & 1 \\ -4 & 9 & 1 \\ 1 & 1 & 4 \end{pmatrix}.$$

(a). Find eigenvalues and eigenvectors of M. [Do it by hand. But check your work by machine if you wish.]
(b). Verify that eigenvectors for distinct eigenvalues are orthogonal.

5. Exercise 8.21. [Be sure to demonstrate that the U you construct is unitary.]

6. Show that

$$\left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \right\}$$

is a basis of $\mathbb{R}^3$. [That is, show that this set is linearly-independent and that it spans.]

7. **Exercise 12.14**. [I know that the hints give the main results. Therefore your solution must clearly show how to get to these results.]

8. **Exercise 13.13**. [Same comment applies as in **7** above.]

9. **Exercise 14.8**.

10. **Exercise 15.14**.

11. **Exercise 16.9(a)**.