Assignment #5

Due *Thursday, 13 October, 2005.*

I. We are now starting to do lots of ODE problems, and for a while it will all be review of Math 302 material. Therefore you might want to find a textbook for that course, for instance

- Boyce & DiPrima, *Elementary Differential Equations*, or
- Nagle & Saff, *Fundamentals of Differential Equations*,

or even the versions of these texts which address boundary value problems. (See me if you seek a free version of such an ODE text—I may be able to help you.) Our current text Riley, Hobson, & Bence is sufficiently self-contained on the topic of ODEs, but, as a student, I always wanted to "get a second opinion." In particular, I have never trusted the authors of textbooks about the relative importance of topics; all authors, myself included, have pet topics/techniques. (*See if you can guess what mine are!*)

One mathematical prerequisite for the use of Laplace transforms is the pre-calculus technique of *partial fractions*. This explains why I have assigned material from chapter 1 below.

From the same chapter I have also assigned a couple of easy proofs by induction. That proof technique will come up several times, including when we talk about orthogonal polynomials. There is a model proof by induction on pages 31–32 of Riley, Hobson, & Bence, and here is another model proof in an oh-so-slightly different style:

$$\text{For each integer } n \geq 0, \quad \int_0^\infty x^n e^{-x}\,dx = n!.$$

[*We want to prove an infinite list of propositions (statements), namely the proposition "$\int_0^\infty x^n e^{-x}\,dx = n!$" for each of $n = 0, 1, 2, \dots$. The principle of mathematical induction says that we can prove* all *of these propositions by proving the $n = 0$ proposition and then proving that the n proposition implies the $n + 1$ proposition.*]

Proof. When $n = 0$, we see that

$$\int_0^\infty x^0 e^{-x}\,dx = \int_0^\infty e^{-x}\,dx = -e^{-x}\big|_0^\infty = -(0 - 1) = 1 = 0!,$$

so the $n = 0$ proposition is true. Now we integrate-by-parts on the $n + 1$ integral:

$$\int_0^\infty x^{n+1} e^{-x}\,dx = -x^{n+1}e^{-x}\big|_0^\infty - \int_0^\infty (n+1)x^n(-e^{-x})\,dx = 0 + (n+1)\int_0^\infty x^n e^{-x}\,dx.$$

If the nth proposition holds, that is, if $\int_0^\infty x^n e^{-x}\,dx = n!$, then

$$\int_0^\infty x^{n+1} e^{-x}\,dx = (n+1) \cdot n! = (n+1)!$$

Mathematical induction now says that all cases of the proposition are true. □

[*Note that the integration-by-parts applies when $n \geq 0$. If one rewrote the proof showing "$(n-1)$ proposition implies n proposition" then one would have to note that the integration-by-parts only applied when $n \geq 1$. This is a minor reason to prefer the way I did it...*]

II. Do exercises

1.15(a), 1.16, 1.18(a), 1.21, 1.22,
13.21, 13.22, 13.24, 13.26, 13.28(ab), 14.2, 14.4, 14.5(ab).