Math 422 Introduction to Complex Analysis (Bueler) 15 March, 2016

Assignment #7

Due Wednesday, 30 March 2016

I will not be lecturing on Monday 3/21, Wednesday 3/23, and Friday 3/25.[1] This Assignment and some online slides replace the lecture.

Please read Sections I.6, IV.1 and IV.2 in the textbook. View the slides, which complement the text, at

`bueler.github.io/M422S16/pathintegrals.pdf`

I will grade the circled Exercises and Problems P2 and P3. Those Problems (second page) introduce Newton's method using complex numbers. They are self-contained, but I'll return to their topic in class when I return. Make sure to ask me questions about this Assignment in-class on Monday 3/28!

Section I.6, page(s) 24, Exercises:

1 (a) (b) (c)

[✓ 2 (a) (b) (c)]

Section IV.1, page(s) 106–107, Exercises:

1 (a) (c)

[✓ 2 (a) (c)]

[✓ 3 (a) (c)]

[✓ 4]

[✓ 5]

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Section IV.2, page(s) 109–110, Exercises:

[✓ 1 (a) (b)]

[✓ 2]

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[1] At conference on iterative and Newton methods: `grandmaster.colorado.edu/~copper/2016/`

Problem P2. (a) Consider a differentiable, real-valued function $f(x)$ which is defined on the whole real line. The equation

$$f(x) = 0 \tag{1}$$

is often (usually) impossible to solve by algebraic methods. Newton's idea was to approximate $f(x)$ by a linear function, the tangent-line approximation of $f(x)$ at a point x_k:

$$f(x) \approx L(x) = f(x_k) + f'(x_k)(x - x_k).$$

The idea is to guess at x_k and solve $L(x) = 0$ instead of (1). A linear equation is always easy to solve by algebra we know! The solution of $L(x) = 0$ should be closer to the solution of (1).

Based on the above ideas, derive this well-known formula for *Newton's method*:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}. \tag{2}$$

(b) The value x_{k+1} computed from (2) is treated as an improved guess. One starts with an initial guess x_0 to get the *Newton iteration* $x_1, x_2, \dots,$ going. An example is appropriate. After confirming informally that you do not know how to solve

$$\cos(x) = x^3 \tag{3}$$

by algebra, solve it very accurately by Newton's method; use a calculator or computer. (*Hints: Sketch the two sides of* (3) *well enough to guess at an initial* x_0. *Write* (3) *in the form of* (1) *and set up* (2). *Show me your work, and give the first 12 digits of the solution to* (3).)

Problem P3. Newton's method does not have to be done with real numbers. It works with complex numbers, although the geometric interpretation is less clear. For this problem, assume $f(z)$ is analytic and use the same Newton's method formula, starting with a complex initial guess z_0:

$$z_{k+1} = z_k - \frac{f(z_k)}{f'(z_k)} \tag{4}$$

Again we need an example. Let $f(z) = z^4 + z^2 - 2$. Solve $f(z) = 0$ by hand; yes it is easy to factor and find all roots by hand! Now try each of the following initial guesses and do enough Newton iterations to confidently-determine which root of $f(z) = 0$ is the one that the Newton iterates are converging to:

$$z_0 = 2, \qquad z_0 = 1 + 2i, \qquad z_0 = 1 + i.$$

(*Hints: A calculator or computer is required, of course. In* MATLAB *it is easy to do these iterations at the command line. You might define "anonymous functions" for $f(z)$ and its derivative $f'(z)$, and then write a loop at the command line.*)