NAME: ____________________

MATH 422 Complex Analysis (Bueler) 4 April, 2008

Midterm Exam # 2

100 points total. You have 60 minutes.

1. (*10 pts*) Suppose $f(z) = u(x, y) + iv(x, y)$ is analytic in an open set. Use the Cauchy-Riemann equations to show that the real part u is harmonic, that is, $u_{xx} + u_{yy} = 0$. What assumptions, if any, are necessary for your derivation to be correct?

2. (*8 pts*) Compute $\log(-i)$. (*Recall that "*$\log$*" is multi-valued.*)

3. (*8 pts*) Explain why

$$\int_{-C} f(z)\,dz = -\int_C f(z)\,dz.$$

4. (*8 pts*) Show that $\exp(z)$ is real if and only if $\operatorname{Im} z = n\pi$ for some integer n.

5. **(a)** (*5 pts*) Define "$\cos z$" if z is a complex number.

(b) (*5 pts*) Show that $\cos z = \cos x \cosh y - i \sin x \sinh y$ if $z = x + iy$.

6. **(a)** (*8 pts*) Show that $f(z) = \exp(\bar{z})$ is not analytic at any point of the complex plane.

(b) (*9 pts*) Let C be the contour given by the parameterization $z(t) = t + i(1 - t)$ for $0 \le t \le 1$. For the function $f(z)$ in part **(a)**, compute

$$\int_C f(z)\, dz.$$

7. **(a)** (*5 pts*) State the definition of "z^c" for z and c complex numbers (and $z \neq 0$).

(b) (*8 pts*) Find the principal value of $(1+i)^{4i}$.

(c) (*8 pts*) Show that $(z^c)^n = z^{cn}$ if $n = 1, 2, 3, \ldots$.

8. **(a)** (*5 pts*) Find an antiderivative of $f(z) = 1/z^2$. Is this antiderivative multi-valued? Can an antiderivative ever be multi-valued according to the definition in the textbook?

(b) (*10 pts*) Suppose C is the circle $z = e^{i\theta}$, $-\pi \le \theta \le \pi$. Compute

$$\int_C \frac{dz}{z^2}$$

Extra Credit. (*3 pts*) By writing $z = re^{i\theta}$, show that

$$\log(z^{1/n}) = \frac{1}{n}\,\log(z)$$

if $n = 1, 2, 3, \ldots$.

EXTRA SPACE