NAME: ______________________

MATH 422 Complex Analysis (Bueler) 5 March, 2008

Midterm Exam # 1

100 points total. You have 60 minutes.

1. **(a)** (*5 pts*) For a function $f(z) = u(x, y) + iv(x, y)$, state the Cauchy-Riemann equations.

(b) (*15 pts*) Show that the function $f(z) = e^{-x} \cos y - ie^{-x} \sin y$ is complex differentiable at every point in the plane. Explain what facts or theorems you are using.

3. **(a)** (*15 pts*) Draw a decent sketch the set of points z, in the complex plane, given by the inequalities $1 < |z + i| < 2$. (*Shade the set for clarity, and indicate several specific points in the complex plane, to give a scale.*)

(b) (*5 pts*) Define what it means for a set of complex numbers to be a *domain*. (*Give the new definition, not the definition of* domain of a function.)

(c) (*5 pts*) Is the set in part **(a)** open, closed, or neither? Is the set in part **(a)** a domain?

4. **(a)** *(10 pts)* Find the real and imaginary parts u, v of the function $f(z) = \bar{z}^3$.

(b) *(10 pts)* For the function f defined in part **(a)**, does $f'(i)$ exist? Explain, and state what facts or theorems you are using.

(c) Extra Credit. *(3 pts)* Show that, for the function f defined in part **(a)**, $f'(0)$ does not exist.

5. (*15 pts*) Use the definition of the limit to prove that

$$\lim_{z \to z_0} \operatorname{Re} z = \operatorname{Re} z_0.$$

6. (*10 pts*) Define $\operatorname{Arg} z$.

7. (*10 pts*) Reduce this to a real number:

$$\frac{4i}{(1-i)(2-i)(3-i)}$$

EXTRA SPACE