Final Assignment/Exam

DUE *Friday 9 May, 2008* at 5pm

(in my box in Chapman 101 or under my door at Chapman 301)

Rules. You *may* use written or online references for basic facts, though most of these facts can be found in the textbook. You *may* use MATLAB and other calculation technology to produce plots and compute numbers. You *must* provide references to what you found and where, and to what you did if it involved nontrivial computer use.

You *may not* talk to, or communicate with by any method, any person other than *me* about the content of this exam.

Please come to me with questions, including questions about these rules. I will be genuinely helpful and not obscure. Email `ffelb@uaf.edu` or call 474-7693.

1. (*8 points*) Exercise #7 on page 13 of BROWN & CHURCHILL.

2. (*8 points*) Exercise #1 on page 96 of BROWN & CHURCHILL.

3. (*8 points*) Show that $f(z) = \cos(z^2)$ is entire.

4. **(a)** (*6 points*) Show $u(x, y) = x \sinh x \cos y - y \cosh x \sin y$ is harmonic.

(b) (*6 points*) Find a harmonic conjugate v of u in part **(a)**. Then describe *all* harmonic conjugates of u.

5. (*8 points*) Exercise #11 on page 80 of BROWN & CHURCHILL.

6. (*8 points*) Exercise #11 on page 104 of BROWN & CHURCHILL.

7. (*10 points*) Give two Laurent series expansions in powers of z for the function

$$f(z) = \frac{z+1}{z^3(1-z)}.$$

(*Hint*: Compare to exercise #4 on page 198.)

8. Let C be the closed contour, forming a triangle, with parameterization

$$z(t) = \begin{cases} t, & 0 \leq t \leq 1, \\ (2-t) + (t-1)i, & 1 \leq t \leq 2, \\ (3-t)i, & 2 \leq t \leq 3. \end{cases}$$

(a) (*6 points*) Compute

$$\int_C z^3 \, dz.$$

(*Hint*: You don't need to worry about the parameterization for this part.)

(b) (*6 points*) Compute

$$\int_C |z|^2 \bar{z} \, dz.$$

(*Hint*: This time, you do.)

9. Let C be the positively-oriented circle of radius 2. Compute

$$\int_C \frac{dz}{(z-1)(i+z)}$$

by three steps.

(a) (*6 points*) Apply partial fractions, i.e. find A and B:

$$\frac{1}{(z-1)(i+z)} = \frac{A}{z-1} + \frac{B}{z+i}$$

(b) (*6 points*) Choose smaller circles C_1 and C_2 and apply the principle of deformation of paths so that

$$\int_C \frac{dz}{(z-1)(i+z)} = \int_{C_1} \frac{dz}{(z-1)(i+z)} + \int_{C_2} \frac{dz}{(z-1)(i+z)}.$$

(c) (*6 points*) Actually compute the integrals over C_1 and C_2.

10. (*8 points*) Using the MacLaurin series for e^z, show that

$$e^{\cos\theta} \cos(\sin\theta) = 1 + \cos\theta + \frac{1}{2}\cos(2\theta) + \frac{1}{3!}\cos(3\theta) + \ldots$$

Note this is a Fourier cosine series on $-\pi \leq \theta \leq \pi$. (*Hint*: Substitute $z = e^{i\theta}$.)