Assignment # 7

Due Wednesday, 16 November, 2011 at start of class.

For each of the following initial/boundary value problems, find the solution $u(x,t)$ for $0 < x < 1$ and $t > 0$.

I *suggest* that you think of these calculations as the following four steps, which form what the textbook appropriately calls "Fourier's method":

(1) Separate the variables $u(x,t) = X(x)T(t)$, and then state the ODEs solved by $X(x)$ and $T(t)$.
(2) Fully state and fully solve the eigenproblem for $X(x)$. This gives the infinite lists λ_n and $X_n(x)$.
(3) Solve the corresponding problems for $T_n(t)$. Thereby state the general solution of the PDE and BCs, which is an infinite sum with unknown coefficients.
(4) Use orthogonality to find those coefficients from the IC(s).

P3.

PDE	$u_{tt} = 4u_{xx}$
BCs	$u(0,t) = 0$ $u(1,t) = 0$
ICs	$u(x,0) = \sin(3\pi x)$ $u_t(x,0) = 0$.

P4.

PDE	$u_t = u_{xx} - u$
BCs	$u_x(0,t) = 0$ $u_x(1,t) = 0$
IC	$u(x,0) = x^2$.

P5.

PDE	$u_t = u_{xx}$
BCs	$u_x(0,t) = 5$ $u(1,t) = 0$
IC	$u(x,0) = 2x$.