Assignment # 5

Due Friday, October 28, 2011 at start of class.

***Lesson 1*, # 1.** Do all parts (a)–(d), and add one more:

(e)

$$H_t = (H^5 |H_x|^2 H_x)_x$$

In classifying, please do not apply terms for a linear equation to a nonlinear equation.

***Lesson 1*, # 2.**

***Lesson 1*, # 5.**

***Lesson 2*, # 2.**

***Lesson 5*, # 1.**

***Lesson 5*, # 2.**

P1. Show that the wave equation

$$u_{tt} = c^2 u_{xx}$$

is *linear* by showing that if $u_1(x,t)$ and $u_2(x,t)$ are solutions of the wave equation, and if α_1, α_2 are real numbers, then

$$u(x,t) = \alpha_1 u_1(x,t) + \alpha_2 u(x,t)$$

is also a solution.

P2. *Figure 3.4 on the top of page 22 should suggest a connection to the vector calculus we have recently experienced. That figure motivated me to write this exercise.*

Suppose that for every closed surface S the temperature $u(x,y,z,t)$ satisfies

$$\iiint_V \rho_0\, c_0\, \frac{\partial u}{\partial t}(x,y,z,t)\, dV = -\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

where V denotes the volume enclosed by S. Here ρ_0 is the density of a material (a positive constant for simplicity) and c_0 is called the specific heat capacity (also a positive constant for simplicity). Note this equation says that the only way thermal energy can get into or out of S is by a flux $\mathbf{F}$. Also assume that *Fourier's law* is true, namely that the flux $\mathbf{F}$ is itself a gradient,

$$\mathbf{F} = -k_0 \nabla u$$

where ∇u is the usual spatial gradient and k_0 is the conductivity (again a positive constant for simplicity). Derive the three-dimensional heat equation,

$$\frac{\partial u}{\partial t} = \alpha^2 \nabla^2 u$$

where α^2 is the constant

$$\alpha^2 = \frac{k_0}{\rho_0 c_0}.$$

This last constant is called the *diffusivity*, and it is essentially geometric, unlike ρ_0, c_0, k_0 which all have to do with measurable properties of a material.