# On proving and writing proofs.

If you are asked to "show that ..." or "prove that ..." then you must clearly understand the range of cases you are addressing (this is essentially the same as understanding what assumptions you may make) *and* you must understand the conclusion you wish to draw. Then you must make an appropriately general, precise, and complete argument. That is, you need to *prove*.

On the other hand, a proof is just a careful argument that goes with the complete logical understanding of a situation. That's math.

*So* I recommend the style of proof below in order to give you a template for such a careful and complete argument. You are not obliged to use the template, but you must make the careful and complete argument, and it must have reflect the kinds of "understanding" described above.

Here is **1–2 #4** written out in a good style:

Exercise 1–2 # 4.

**To Show:** Let $\alpha : I \to \mathbb{R}^3$ be a parameterized curve. Let $v \in \mathbb{R}^3$ be a fixed vector. Assume $\alpha'(t)$ is orthogonal to $v$ for all $t \in I$ and assume that $\alpha(0)$ is also orthogonal to $v$. Then $\alpha(t)$ is orthogonal to $v$ for all $t \in I$.

**Proof**. Describe $\alpha$ by its components: $\alpha(t) = (x(t), y(t), z(t))$. Similarly for $v$: $v = (a, b, c)$. We know

$$ax'(t) + by'(t) + cz'(t) = 0 \qquad \text{for all } t \in I \tag{1}$$

and we know

$$ax(0) + by(0) + cz(0) = 0.$$

By integrating (1) we get

$$ax(t) + by(t) + cz(t) = d \qquad \text{for all } t \in I$$

for some constant $d$. But

$$0 = ax(0) + by(0) + cz(0) = d.$$

So

$$ax(t) + by(t) + cz(t) = 0 \qquad \text{for all } t \in I.$$

Thus $\alpha(t)$ is orthogonal to $v$ for all $t \in I$. $\square$

Note that:

- what I assume is clearly stated;
- what I intend to prove (the claim) is clearly stated (after "Then");
- the proof is separated from the claim, and its beginning and end are indicated.

The above style helps when determining if an argument does or does not show/prove/explain the claim. For instance, if you find you cannot prove the most general statement, but you can prove something which (for instance) has stronger assumptions but the same conclusion, then that situation is clear. And you will get an appropriate amount of credit.