

Review Guide for In-Class Midterm Exam I on *Monday, 3 October 2016*

The first Midterm Exam will cover Chapters 1, 2, 3, 4, 5, and 6 in Sutherland, *Introduction to Metric & Topological Spaces*, 2nd edition. See below for excluded material that will *not* be on the Exam. The Exam is *closed-book and closed-notes*. The problems will be of these types: state definitions, prove propositions which follow reasonably directly from the definitions or from known facts (below), give examples with certain properties, or describe or illustrate (sketch) concepts/examples.

I encourage all students to get together and work through this Review Guide. Be honest with yourself about what you do and don't know, and talk it through with your friends!

This first in-class Exam requires you to become comfortable with the definitions. The questions will be straightforwardly-stated, such as "define bounded set for a metric space" or "prove that if $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous then $|f|$ is continuous" or similar. I am aware that you will have no book or notes during the exam, and so I will never ask you to "prove theorem 4.23" or anything like that.

Excluded material. The following material will *not* be on the exam:

- pages 18–19, Chapter 4, on least upper bound (sup) and greatest lower bound (inf)
- page 44, Chapter 5, Examples 5.11 and 5.12
- pages 64–65, Chapter 6, on limit points
- pages 69–72, Chapter 6, on equivalent and Lipschitz-equivalent metrics and isometry

Definitions. Be able to state and use the definition:

- on page 5:

$$\bigcup_{i \in I} A_i, \quad \bigcap_{i \in I} A_i, \quad A \times B$$

- graph of f (page 6)
- composition of functions (page 6)
- injective (one-to-one), surjective (onto), bijective (page 6)
- image $f(A)$ and preimage (inverse image) $f^{-1}(C)$ (page 9)
- f invertible (page 13) and inverse function f^{-1} (page 14)
- real sequence (s_n) converges to $l \in \mathbb{R}$ (page 21)
- $\lim_{x \rightarrow a} f(x) = L$ for $f : \mathbb{R} \rightarrow \mathbb{R}$ (page 25)
- for $f : \mathbb{R} \rightarrow \mathbb{R}$ (page 28) or $f : \mathbb{R}^n \rightarrow \mathbb{R}$ (page 38): continuous at a , continuous
- metric $d : X \times X \rightarrow \mathbb{R}$ (page 38) and metric space (X, d) (page 39)
- discrete metric on any X (page 41)
- d_1, d_2, d_∞ on \mathbb{R}^n (pages 40 and 42) and on $X \times Y$ (page 43)
- (A, d_A) , a metric subspace for $A \subseteq X$ (page 43)
- for (X, d) a metric space:
 - $S \subseteq X$ bounded (page 50) and $f : S \rightarrow X$ bounded (page 50)
 - open ball $B_r(x_0)$ (page 51)

- $U \subseteq X$ open in X (page 54)
- $f : X \rightarrow Y$ continuous (page 55)
- $V \subseteq X$ closed in X (page 61)
- for $A \subseteq X$, where (X, d) is a metric space:
 - closure \bar{A} (page 63)
 - A is dense in X (page 63)
 - interior $\overset{\circ}{A}$ (page 66)
 - boundary ∂A (page 67)
- for (X, d) a metric space:
 - sequence (x_n) converges to x (for $x, x_n \in X$; page 68)
 - sequence (x_n) is Cauchy (for $x_n \in X$; page 68)

Propositions. Understand and remember these as facts. You can use these facts as needed in proving other propositions, but mention it if so (e.g. “because a function is invertible if and only if it is bijective”). Be *able* to prove each of these, if requested, unless it is otherwise noted.

- De Morgan’s laws (page 5)
- proposition 3.7 (page 11)
- propositions 3.18 and 3.19 (pages 13–14)
- proposition 4.6, corollary 4.7, remark 4.8 (pages 19–20) [*Proofs will not be requested.*]
- theorem 4.18 (page 24) [*Proof will not be requested.*]
- proposition 4.20 (page 25)
- propositions 4.31, 4.32, 4.33 (pages 30–32)
- lemma 5.15 (page 46) [*Proof will not be requested.*]
- propositions 5.17, 5.18, 5.19 (pages 49)
- proposition 5.26 (page 51)
- proposition 5.37 (page 55)
- propositions 5.39, 5.41 (pages 56–57)
- propositions 6.3, 6.4, 6.5 (pages 61–62)
- proposition 6.11 (page 63) [*Proofs of difficult parts will not be requested.*]
- proposition 6.21 (page 66) [*Proofs of difficult parts will not be requested.*]
- proposition 6.24 (page 67)
- proposition 6.28 (page 68)

Examples, counter-examples, and sketches. Be able to give an example, sketch, or Venn diagram as appropriate.

- images and preimages (pages 9–13)
- graphs and inverse functions (pages 10–15)
- limit of a real sequence (page 22) or real function (pages 25–26)
- continuity of a real function (page 29)
- triangle inequality in Euclidean space (page 39)
- open balls in \mathbb{R}^2 for metrics d_1, d_2, d_∞ (pages 42 and 52)