Assignment #2

Due Friday, 16 September 2016, at the start of class

Please *carefully* read Chapter 4 in the textbook, Sutherland, *Introduction to Metric and Topological Spaces*.

Note that I am currently emphasizing the concepts of *limit* and *continuous* from Chapter 4. Later in the course we will return to the concepts of *least upper bound*, *greatest lower bound*, *sequences*, and the *completeness property of* $\mathbb{R}$, which the author has put earlier in Chapter 4. But please do read all of Chapter 4 now!

Do the following exercises.

Chapter 3, pages 15, Exercises:

3.7

3.8 (You are asked to prove two things. I suggest you just write separate proofs for simplicity.)

Chapter 4, pages 33–35, Exercises:

4.12 (*Hint*: In my version, both f and g are defined on all of $\mathbb{R}$, but $f(x)$ is very trivial.)

4.13

4.14

4.15

Problem P2. Prove, by elementary means as shown in class, namely *without* using the continuity of any function, that

$$\lim_{x\to-1} \frac{2x+2}{x^2-6x-7} = -\frac{1}{4}.$$

Problem P3. Prove, by direct use of Definition 4.29, that

$$f(x) = \frac{1}{x^2+2}$$

is continuous at $a = 1$. (*That is, start the proof with "let* $\epsilon > 0$*" and show constructively that* δ *exists so that* $|x - a| < \delta$ *implies* $|f(x) - f(a)| < \epsilon$*. A hint about this particular function is that you* do not *need to factor* $x^2 + 2$ *but that you* probably do *want to write* x^2 *in terms of the distance from* $a = 1$.)