Math 401 Intro Real Analysis (Bueler) Name: _____

November 13, 2013

Midterm # 2

[100 points total]

- **No notes. No book.**
- **Please write clearly and in complete sentences.**
- **Please mark the start and end of proofs in the standard way.**
- **Please cross-out work that should not be graded.**
- **So as to save time, you *do not* need to restate what you are proving.**

1. (a) [5 points] Suppose (s_n) is a real sequence. Define $\limsup s_n$.

(b) [10 points] Suppose (s_n) is a bounded real sequence. Show $\liminf s_n$ exists, that is, it is a real number. (*State any relevant theorems.*)

2. [15 points] Show that if $s_n \to s$, where s is a real number, then (s_n) is Cauchy.

3. [10 points] Suppose $f : \text{dom}(f) \to \mathbb{R}$ is a real function with domain $\text{dom}(f)$. Suppose $x_0 \in \text{dom}(f)$. Define *f is continuous at x_0*.

4. [15 points] Suppose $a, r \in \mathbb{R}$ and $|r| < 1$. Show that

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

(*Start from the beginning on this proof. Don't prove it by saying "Because the series is geometric ..." or something like that.*)

5. (a) [5 points] Write $0.575757\cdots = 0.\overline{57}$ as a rational number, that is, as a ratio of integers.

(b) [5 points] Write $0.\overline{57}$ as a convergent geometric series $\sum_{n=0}^{\infty} ar^n$ with nonzero r. (*Specifically*, give a and r.)

6. Do the following series converge or diverge? In either case, show this, stating any relevant theorems and/or "Tests".

(a) [10 points]

$$\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$$

(b) [10 points]

$$\sum_{n=0}^{\infty} \frac{2}{n! + 7}$$

7. [15 points] Show that if a series converges then its terms go to zero.