Assignment #2

Due Monday 23 September, 2013 at the start of class

Please read Sections 3, 4, 5, and 6 of the textbook *Elementary Analysis*. Then do the following exercises and turn them in on paper. The one circled problem on your paper is the one you should do in LaTeX and email to me.

Exercise E1. In addition to the principle of mathematical induction (PMI), which is essentially the textbook's axiom N5 for the natural numbers, *complete induction* (or *strong induction*), says that to prove P_n for all n we may show

- P_1 is true, and
- if all of $P_1, P_2, \dots, P_n$ are true then P_{n+1} is true.

This form of induction should be easier to use because it is logically "stronger" than PMI. That is, it allows you to prove the same consequence ("P_n is true for all n") from a logically "weaker" statement ("if all of $P_1, P_2, \dots, P_n$ are true then P_{n+1} is true") which allows you to assume that more hypotheses are true. In fact complete and ordinary mathematical induction are equivalent.

Let $\phi = (1+\sqrt{5})/2$ and $\psi = (1-\sqrt{5})/2$. Use complete induction to prove that if F_n are the Fibonacci numbers, which satisfy $F_1 = 1$, $F_2 = 1$, and

$$F_n = F_{n-1} + F_{n-2}$$

for all natural numbers $n \ge 3$, then

$$F_n = \frac{\phi^n - \psi^n}{\sqrt{5}}$$

for all natural numbers $n \ge 1$.

Exercise 3.1.

Exercise 3.3.

Exercise 3.4.

Exercise 3.5.

Exercise 3.6.

Exercise 4.5.

Exercise 4.6.

Exercise 4.7.

Exercise 4.10.