Assignment #1

Due Friday 13 September, 2013 at the start of class

Please read Sections 1 and 2 in Chapter 1 of the textbook *Elementary Analysis: The Theory of Calculus*, 2nd ed., by Ken Ross. The first problem on this Assignment is for familiarizing you with LaTeX. For getting started on LaTeX, and installing it on your own computer, see the Math 401 class webpage:

www.dms.uaf.edu/~bueler/Math401F13.htm

Exercise 1.1. For all students, please LaTeX this proof. Start with the `.tex` file at

www.dms.uaf.edu/~bueler/Sample401Proof.tex

Modify it so it is *just like* the version I did on the back of this assignment. Thus there is no mystery about the proof! This is just a LaTeX exercise. Give the new `.tex` file a new name and email it to me at `elbueler@alaska.edu`.

The remainder of this assignment should be turned in on paper:

Exercise 1.2.

Exercise 1.5.

Exercise 1.6.

Exercise 1.9.

Exercise 2.2.

Exercise 2.4.

Exercise 2.5.

Exercise 2.7.

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Exercise 1.1. $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ for all positive integers n.

Note that the nth proposition is

$$P_n : \quad 1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

Proof. Proposition P_1 asserts $1^2 = \frac{1}{6}(1)(2)(3)$ which is true. This is the base for our induction.

Suppose P_n. We wish to show that P_{n+1} is true, namely

$$1^2 + 2^2 + \cdots + n^2 + (n+1)^2 = \frac{1}{6}(n+1)((n+1)+1)(2(n+1)+1).$$

But, starting with the left-hand quantity,

$$\begin{aligned}
1^2 + 2^2 + \cdots + n^2 + (n+1)^2 &= \frac{1}{6}n(n+1)(2n+1) + (n+1)^2 \\
&= \frac{1}{6}(n+1)\Big[n(2n+1) + 6(n+1)\Big] \\
&= \frac{1}{6}(n+1)\left[2n^2 + n + 6n + 6\right] \\
&= \frac{1}{6}(n+1)\left[2n^2 + 7n + 6\right] \\
&= \frac{1}{6}(n+1)\Big[(n+2)(2n+3)\Big] \\
&= \frac{1}{6}(n+1)((n+1)+1)(2(n+1)+1).
\end{aligned}$$

The first of the above equalities uses P_n. In the second equality we factor $\frac{1}{6}(n+1)$, and after that we collect terms. By induction we have proven P_n for all positive integers n. □