Name: ______________________

Math 314 Linear Algebra (Bueler) Due Wednesday 9 May 2007 at 10:15am

Final: Matlab Part

Total of 30 points.

Due *Wednesday 9 May* at start of in-class Final Exam.

Please produce a human-readable result. Print out only the correct inputs and results, and not trial and error (when such occurred). You may want to do the work at the Matlab prompt ">>" and then cut and paste only the good parts into an editor, and print that out. Finally, please attach this paper to the front of your output. Remember to write your name at the top.

1. (a) (*3 pts*) Use Matlab to find the determinant of this matrix:

$$A = \begin{bmatrix} 4 & 11 & -12 & -5 & 2 \\ 10 & -8 & -6 & 1 & 3 \\ -9 & -7 & 0 & 7 & 9 \\ -3 & -1 & 6 & 8 & -10 \\ -2 & 5 & 12 & -11 & -4 \end{bmatrix}.$$

(b) (*5 pts*) Consider Theorem 6.10 on page 374 of the textbook. Using Matlab, check that the following formula for a determinant of a 5×5 matrix $A = [a_{ij}]$ is true on the matrix in part **(a)**:

$$\det(A) = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} + a_{14}A_{14} + a_{15}A_{15}$$

Note that A_{ij}, the *cofactor*, is the $(-1)^{i+j}$ times the determinant of the 4×4 *matrix* found from A by deleting row i and column j.

2. (*7 pts*) The columns of A in the previous problem span a subspace W of R^5. I claim the following vector $\mathbf{w}$ is in that subspace:

$$\mathbf{w} = \begin{bmatrix} 37 \\ 27 \\ 2 \\ -43 \\ -23 \end{bmatrix}.$$

Show that this is true, that is, show that $\mathbf{w}$ is in W by writing $\mathbf{w}$ as a linear combination of the columns of A.

3. **(a)** (*8 pts*) Consider the 7×7 matrix

$$M = \begin{bmatrix} m_{ij} \end{bmatrix} = \begin{bmatrix} 0.6 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0.4 \end{bmatrix}$$

This matrix comes from a Markov chain like the sunny/cloudy/rainy example done in class. The "Markov chain" is the equation

$$\mathbf{v}_{n+1} = M\,\mathbf{v}_n.$$

This equation computes the new vector of probabilities $\mathbf{v}_{n+1}$ from the old (or current) probabilities $\mathbf{v}_n$.

In this case each entry x_j of $\mathbf{v}_n = [x_1 \ \dots \ x_7]^\top$, for $j = 1, \dots, 7$, is the probability that a particle is currently at the jth location in this row of boxes:

1	2	3	4	5	6	7

The probabilities m_{ij}, which are the entries of M, are the probabilities of moving to location j if one is currently in location i. This example would usually be named a *biased drunkard's (random) walk*, which surely does not sound like a politically correct name.

Start with the following two vectors

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

as $\mathbf{v}_0$. In each case, use MATLAB to compute $\mathbf{v}_n$ for large enough n so that the results, starting with the two different $\mathbf{v}_0$, nonetheless agree to two digits for each entry. Report the vector you get; since the two results agree to two digits I will treat it like one vector. Interpret it in terms of the random walk description.

(b) (*7 pts*) Use MATLAB's `eig` to compute the eigenvalues and eigenvectors of M. Identify the eigenvector which corresponds to the vector you computed in part **(a)**. In particular, divide the vector found from `eig` by the sum of its entries to get a vector which agrees to two digits with what you got in **(a)**.