# Worksheet: Basic estimation problems for polynomial interpolants

In each problem there are four actions:

**(a)** Write down the full *unsimplified* Lagrange form of the degree $n$ polynomial.
**(b)** Find the derivative of $f(x)$ necessary to compute the remainder term.
**(c)** Write down the remainder term with the ranges for $x$ and $\xi$.
**(d)** Estimate the size of the remainder term.

These steps will be illustrated in an example on the board.

**I.** $f(x) = \sin x$ on $[0, \pi/2]$ using points $x_0 = 0, x_1 = \pi/4, x_2 = \pi/2$.

**(a)**

$p(x) =$

**(b)**

$f^{(n+1)}(x) =$

**(c)**

$R_n(x) =$

**(d)**

$|f(x) - p(x)| = |R_n(x)| \leq$

**II.** $f(x) = \frac{1}{x}$ on $[1, 2]$ using points $x_0 = 1, x_1 = 2$.

**(a)**

**(b)**

**(c)**

**(d)**

**III.** $f(x) = e^{-x}$ on $[0, 1]$ using points $x_0 = 0, x_1 = 1/3, x_2 = 2/3, x_3 = 1$.

**(a)**

**(b)**

**(c)**

**(d)**