Worksheet: Order of accuracy for basic integration rules

In each problem, determine the order of accuracy of the rule as an approximation of $\int_{-1}^{1} f(x) dx$.

I. The trapezoid rule:

$$\int_{-1}^{1} f(x) \, dx \approx f(-1) + f(+1)$$

II. The midpoint rule:

$$\int_{-1}^{1} f(x) \, dx \approx 2f(0)$$

III. Simpson's rule:

$$\int_{-1}^{1} f(x) \, dx \approx \frac{1}{3} \left(f(-1) + 4f(0) + f(+1) \right)$$

IV. A weird rule (*to be named later* ...):

$$\int_{-1}^{1} f(x) \, dx \approx f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$