Worksheet: Suppose IEEE had a *binary12* standard ...

Suppose that the IEEE 754 standard discussed in Chapter 5 of the textbook had a 12 bit version. It might look like this:

s	e_1	e_2	e_3	e_4	b_1	b_2	b_3	b_4	b_5	b_6	b_7

If 12 bits were organized this way they could represent the number

$$x = (-1)^s \, (1.b_1 b_2 b_3 b_4 b_5 b_6 b_7)_2 \; 2^{(e_1 e_2 e_3 e_4)_2 - 7_{10}}$$

Note the exception cases:

- exponent bits $(0000)_2$ define the number zero or subnormal numbers
- exponent bits $(1111)_2$ define the other exceptions: $\pm\infty$ and NaN (*but you can ignor the details of* these *exceptions*)

(a) What is the largest real number that this system can represent? Show the bits.

(b) If we do not consider subnormal numbers, what is the smallest positive number that this system can represent? Show the bits. (*I.e. what is the first normal, representable number to the right of zero.*)

(c) What is the value of "machine epsilon" in this system?

(d) What is the representation of 4? What is the largest representable number which is smaller than 8? Show the bits of each number.

(e) In the interval $[4, 8)$, how many numbers can be represented?

(f) Exactly how many distinct non-exceptional numbers can be represented in this system? (*Include the number zero but exclude subnormal numbers and any exceptions using exponent* $(1111)_2$*, i.e.* $\pm\infty$ *and NaN.*)

(g) Show the bits of one subnormal number.