Worksheet: Numerical integration by 4th-degree polynomials

A. As warm-up, integrate by hand and write the result in an organized way:

$$\int_a^b c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 \, dt =$$

B. Suppose we design a MATLAB function for 4th-degree numerical integration (a.k.a. "quadrature"). The code will approximately compute the number $\int_a^b f(t)\,dt$, for any function f and any interval $[a, b]$ input by the user, by approximating f by a polynomial and then exactly integrating the polynomial.

Decide how you would do each of the following steps; write a line or two of code.

(i) Generate five points $x_0, \dots, x_4$ in $[a, b]$. (*Which points to use is up to you!*)

(ii) Set up the 5×5 Vandermonde matrix for these points and compute the coefficients in the degree 4 polynomial $p(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4$ so that $p(x_j) = f(x_j)$ for $j = 0, 1, 2, 3, 4$. (*Or find the interpolating polynomial by some other means!*)

(iii) Output the exact integral

$$\int_a^b p(t)\,dt$$

as the approximation to $\int_a^b f(t)\,dt$. (*See part* **A**!)

C. Write the code. You may call any MATLAB black boxes you want except for numerical integration methods. For instance, you can use $A\backslash b$, `vander()`, `polyfit()`, or `polyval()`.

```
function z = deg4int(f,a,b)
% DEG4INT Approximates the integral of f(t) on [a,b] using 4th degree
% polynomial interpolation.
% Example:
%     >> deg4int(@(x) sin(x),0,pi)  % exact = 2
```

D. For those groups with a computer at hand, test the above code. For example, try the integrals

$$\int_0^{\pi} \sin(t)\,dt, \qquad \int_{-1}^{2} t^4\,dt, \qquad \int_{-1}^{2} t^5\,dt \qquad \int_{-1}^{2} e^{-t^2}\,dt$$

and compare to the exact answers. Also compare to the built-in integrator `quad()`.

E. Discuss how you might improve this numerical integration method.