Worksheet: Implement $\sin(x)$.

The goal of this worksheet is to write a MATLAB code `mysin.m` which computes $\sin(x)$ for any real number x, in radians, with an accuracy better than 10^{-14}, using only basic arithmetic operations (addition, subtraction, multiplication, and division). The basic idea is to first use symmetries of $\sin(x)$ to reduce to the case where $0 \le x \le \pi/2$. Then use a pre-computed polynomial, of sufficient degree to get the desired accuracy, found using Chebyshev points, on that interval.

A. On a separate sheet, sketch a graph of $y = \sin(x)$ showing multiple cycles.

B. Based on your sketch, explain/discuss why the value of $\sin(x)$ for any x can be found by using a value $y \in [0, \pi/2]$ and computing $\sin(y)$ and then $\sin(x) = \pm \sin(y)$.

C. Write a bit of code corresponding to the answer in **B**. Use the `mod()` function to get started; note `mod(a,b)` is the remainder after dividing a by b. This bit of code will start with x and generate y, but it will also store the sign so that the result is the correct value $\sin(x)$.

D. The Chebyshev points in the interval $[0, \pi/2]$, for $j = 0, 1, 2, \ldots, n$, are

$$x_j = \frac{\pi}{4}(\cos(\pi j/n) + 1).$$

(*Check that these points cover the correct interval.*) It is known that

$$|(x - x_0)(x - x_1) \ldots (x - x_n)| \le \frac{\pi^{n+1}}{2^{3n+1}}$$

for $x \in [0, \pi/2]$. (*This comes from rescaling the bound given near the top of page 193 from* $[-1, 1]$ *to* $[0, \pi/2]$.) Use this fact, and the polynomial interpolation error theorem 8.4.1, to find n so that if $p(x)$ is the polynomial of degree n which satisfies $p(x_j) = \sin(x_j)$ for $j = 0, 1, \ldots, n$ then $|\sin(x) - p(x)| \le 10^{-14}$ on $[0, \pi/2]$.

E. Using the result from **C** and **D** to write the code below, which uses nothing but elementary arithmetic and stored (precomputed) values.

While we are cheating to precompute values of sine at the Chebyshev points, in fact these values could have been computed (laboriously) using Taylor series if no existing codes could compute them. In any case, it is a fact that all computer implementations in current sofware use many such pre-computed values.

```
function z = mysin(x)
% MYSIN  Compute sin(x) for x in radians.  Compare SIN.
```