Assignment #9 (*revised AGAIN*)

Due Thursday, 5 December 2019, at the start of class

This Assignment is based on Chapter 11, a good introduction to solving ordinary differential equations on computers. Please read all of sections 11.1 and 11.2; the latter is very substantial! You can skip section 11.3, but read section 11.4 through page 289; you can skip the rest (pages 290–294). In addition to doing the exercises, I recommend playing with additional `ode45()` and Euler's method examples—the latter using codes you write yourself.

When you turn in homework problems involving MATLAB/OCTAVE, the following two expectations always apply:

1. The commands/code that you ran are shown, along with the results.
2. Minimal paper is used, while still fully answering the question.

Do the following exercises:

CHAPTER 11

- Exercise 1 on page 295.
- Exercise 3 on page 295.

For Exercises 4 and 5, skip Heun's method, but indeed "do the same" for the (explicit) midpoint method defined in section 11.2.3.

- Exercise 4 on page 295.
- Exercise 5 on page 295.

- Exercise 6 on page 295.
- Exercise 15 on page 298.

- Exercise A. Consider the ODE IVP
$$y' = 1 + y^2, \qquad y(0) = 0.$$
Note that $f(t, y) = 1 + y^2$ in this problem.
  (a) Apply the trapezoid method (11.18) to the above ODE. (*You will write down a nonlinear equation for y_{k+1} at each time step, based on knowing y_k from the previous step.*)
  (b) For $y_0 = 0$ and $h = 0.1$, part (a) gives an equation which must be solved to find y_1. Write down this equation. Solve it exactly, but observe that you get two solutions. Which is the one which is the correct step of the trapezoid rule?

(c) In part (b) you have an equation to solve for y_1. Use Euler's method to give a first guess $y_1^{(0)}$ for the solution to that equation. Then do one step of Newton's method to get $y_1^{(1)}$; this will be very close to one of the two values you got in part (b). (*And it is common to accept it as the result of one trapezoid rule step, that is, to not solve the implicit step equation exactly but instead just use a fixed and small number of Newton steps.*)