Assignment #7 (*revised*)

Due Tuesday, 12 November 2019, at the start of class

This Assignment is based on Chapter 8 and 10 material.

By now you should be able to read all of Chapter 8, though you may **skip** understanding the following specific material if you want. I will not ask about it on the Final or the assignments (other than extra credit):

- "barycentric" weights and formulas (pages 183–184),
- "divided differences" (subsection 8.3.1 on pages 187–189), and
- the details of piecewise-cubic-Hermite and cubic-spline interpolation (subsections 8.6.1 and 8.6.2 on pages 200–204).

Please also read sections 10.1 and 10.2.

Remember that when you turn in homework problems involving MATLAB/OCTAVE, the following two expectations always apply:

1. The commands/code that you ran are shown, along with the results.
2. Minimal paper is used, while still fully answering the question.

Do the following exercises:

CHAPTER 8

- Exercise 7 (a) and (c) on pages 208–209. (*Just follow the book's instructions with respect to* `interp1`.)
- Exercise 16. (*To start, take a moment to recall what a* parametric curve *is; perhaps see your calculus book. In this problem you are using spline interpolants to generate a parametric curve* $(x(t), y(t))$ *through data points*.)
- Exercise A.
  - (i) Let $f(x) = \arctan(x)$ and suppose $x_0 = -\frac{1}{\sqrt{3}}, x_1 = 0, x_2 = \frac{1}{\sqrt{3}}, x_3 = 1$. Find the unsimplified Vandermonde, Lagrange, and Newton forms of the interpolating polynomial $p(x)$ such that $p(x_i) = y_i$. (*You may use a computer to solve the linear systems that arise for V. and N. forms, but write the resulting polynomials unsimplified. Of course, simplifying is the only way to check your work for consistency!*)
  - (ii) Use the polynomial interpolation theorem 8.4.1 to estimate $|f(x) - p(x)|$ on the interval $[-1, 1]$. (*For this job you will need a formula for a derivative of* f*, and an estimate of the maximum of this derivative on the interval* $[-1, 1]$*. You may use the internet for these tasks, e.g. Wolfram Alpha, but please make it clear what you are getting from such tools.*)

- Exercise B.
  - (i) As stated on page 193 of the text, it is known that the Chebyshev points $x_j = \cos(\pi j/n)$, for $j = 0, 1, \dots, n$, satisfy
  $$\max_{-1\le x\le 1} |(x-x_0)(x-x_1)\cdots(x-x_n)| \le \frac{1}{2^{n-1}}.$$
  Create plots for $n = 8$ and $n = 20$ which illustrate this fact.
  - (ii) Consider $f(x) = \cos(4x+1)$ on the interval $[-1, 1]$. Use the fact stated in (i), and the polynomial interpolation theorem 8.4.1, to find n so that the interpolating polynomial $p(x)$ using the Chebyshev points satisfies $|f(x) - p(x)| < 10^{-15}$. (*Note that you are not asked to find p itself.*)
- Extra Credit. *Clenshaw-Curtis integration* approximates $\int_{-1}^{1} f(x)\,dx$ by using the Chebyshev points, for some n, to compute an interpolating polynomial $p(x) \approx f(x)$, and then computing the exact integral of p. Write a short code which does this, and test it on several functions for which you know the exact integral. For a bit more extra credit, use the barycentric formulas on pages 183–184. Then see Trefethen, L. N. (2008), *Is Gauss quadrature better than Clenshaw–Curtis?*, SIAM Review 50 (1), 67–87. This paper implements C.–C. integration in 6 lines of MATLAB. This paper convinced everyone that it is better than what we thought for 200 years was the best smooth-function integration technique; why?

CHAPTER 10

- Exercise 1.