Worksheet: Derive formulas for root-finding algorithms

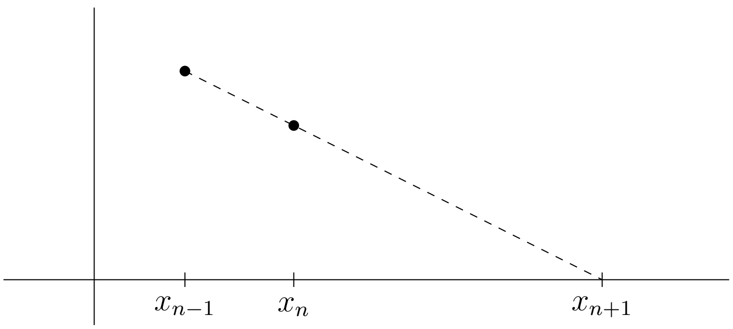
Only two rules:

1. please *do not* use the internet or any book
2. please *do* talk to each other

Secant method. The secant method is like Newton's method but, instead of the tangent line for the current iterate, it uses a secant line based on the *two* most-recent iterates. Use the diagram and elementary algebra to derive the formula:

$x_{n+1} =$

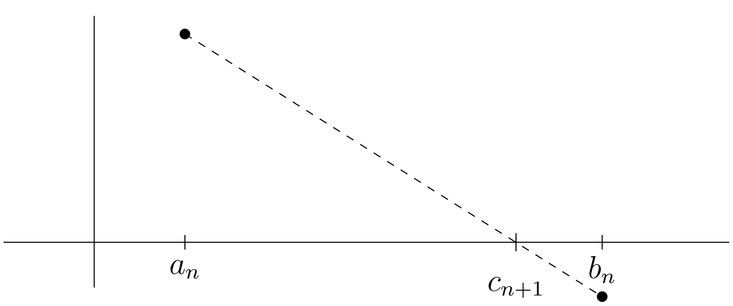
(*On the right will be some formula with* x_n, x_{n-1}, $f(x_n)$, *and* $f(x_{n-1})$.)



Method of "false position." This generalization of bisection uses the current bracket $[a_n, b_n]$, and the points $(a_n, f(a_n))$ and $(b_n, f(b_n))$ themselves, to compute c_{n+1}. (Bisection only uses the bracket to compute $c_{n+1} = (a_n + b_n)/2$, ignoring f-values.) Give the formula:

$c_{n+1} =$

Add a "`if` … `else` …" description to form the next bracket $[a_{n+1}, b_{n+1}]$.



Halley's method. Newton's method replaces a function $f(x)$ by its tangent line. We get the tangent line from the first two terms of the Taylor series: $\ell(x) = f(x_n) + f'(x_n)(x - x_n)$. Then we find x_{n+1} so that $\ell(x_{n+1})$. Halley's method is a fairly-obvious extension: use the first *three* terms of the Taylor series, a quadratic polynomial $p(x)$ and get x_{n+1} as the root of this polynomial. Use this idea, and the sketch, to derive Halley's method:

$x_{n+1} =$

Actually using this formula requires a choice at each step; explain and choose.