Review Guide for In-Class Midterm Exam on Tuesday, 24 October 2017

The Exam is *closed book* and *closed notes* and *no calculator.*

The Midterm Exam on Tuesday 24 October will cover the following sections of the textbook J. Epperson, *An Introduction to Numerical Methods and Analysis*, 2nd edition:

1.1, 1.2, 2.1, 2.2, 2.3, 2.4, 3.1, 3.2, 3.5, 3.6, 3.8, 3.9, 4.1, 4.2, 4.3.

A more detailed list is given below. Some skipped sections make good review reading; see the next page. My goal is to only include topics that have appeared on homework and in lecture, and to only ask questions which are quick to answer (if you are prepared).

Strongly recommended: Please ask questions about Exam content during lecture on Thursday 19 October. Then get together with other students and work through this Review. Be honest with yourself about what you do and don't know. Talk it through and learn!

Sections covered.

- 1.1 Be able to state Taylor's Theorem with Remainder, Theorem 1.1. Be able to apply (use) it for small values of n and for a given $f(x)$, including finding bounds on the remainder. Note we have only used form (1.3) of the remainder. Be able to state and apply the Mean Value Theorem and Intermediate Value Theorem. (*You do not have to memorize any Taylor series. You can skip Theorems 1.4, 1.5, 1.6.*)
- 1.2 Know the definitions of absolute error, relative error, and $+O(h^q)$. For the last, note that if I ask you to "show $\dots +O(h^q)$" then you need to show an inequality that looks like "$|\cdots - \cdots| \le Ch^q$."
- 1.3 Know the definition of machine epsilon. (*The rest of this section is too technical to appear on the exam, except perhaps as extra credit.*)
- 2.1 Be able to use Horner's method to rewrite a polynomial. Be able to write Horner's method as a pseudocode.
- 2.2 Be able to derive difference formulas like (2.1) and (2.4) from Taylor's theorem. Be able to write such formulas using "$+O(h^q)$."
- 2.3 Be able to sketch, write as a pseudocode, and do a couple of steps of Euler's method.
- 3.1 Be able to write the bisection method as a pseudocode, and do a couple of steps on an example. Understand and be able to derive bound (3.2). What advantage does bisection have over Newton's and secant methods?
- 3.2 Be able to derive Newton's method (3.7), justify it with a sketch, and write it as a pseudocode. Be able to derive (3.7) from Taylor's Theorem with $n = 1$. Be able to do a couple of steps on an example. What advantage does Newton's have over bisection and secant?
- 3.5 Understand Theorem 3.2 and its proof. (*You will not need to state it from memory.*) Know the definitions of order of convergence p, superlinear convergence, and quadratic convergence.

3.6 Understand how Theorem 3.3 is derived from Theorem 3.2. (*You will not need to state Theorem 3.3 from memory either. Theorem 3.4 can be skipped.*)

3.8 Be able to derive the secant method (3.28), justify it with a sketch, and write it as a pseudocode. Be able to do a couple of steps on an example. What advantage does secant have over bisection and Newton's?

3.9 Be able to sketch a fixed-point iteration example. Understand and be able to apply Theorem 3.5. (*Theorems 3.6 and 3.7 can be skipped.*)

4.1 Be able to state Theorem 4.1. Be able to compute Lagrange's functions $L_i(x) = L_i^{(n)}(x)$ for a given set of nodes $\{x_j\}$. Be able write down the polynomial $p_n(x)$ by this Lagrange construction, and apply this method for small examples. Be able to set up a linear system for $p_n(x)$ in standard form (Vandermonde construction), and apply this method for small examples.

4.2 Understand Theorem 4.2. Be able to set up a triangular linear system for $p_n(x)$ in Newton's form, and apply this method for small examples. Be able to write Newton's construction, Algorithm 4.1, as a pseudocode.

2.4 Be able to derive (2.11), for example from Lagrange's construction in section 4.1. Understand and be able to apply Theorem 2.1. (*There is no reason to memorize Theorem 2.1, and I will not ask you to prove it.*)

4.3 Understand and be able to state Theorem 4.3 (noting its close analogy to Theorem 1.1). Be able to apply Theorem 4.3 to bound the interpolation error, especially in $n = 1, 2$ cases. Know the definition of $\underline{\|f\|_\infty}$ (on a given interval $I = [a, b]$).

Types of questions. Looking at the above list you will see these types of questions:

(a) state theorems from memory ← *know the name of the Theorem, not the number*
(b) state definitions from memory
(c) apply theorems, which often means showing an inequality
(d) sketch the idea/derivation of a method
(e) do calculations which apply a method
(f) write pseudocodes
(g) do a couple of steps of an iteration

A good review exercise is to go through the "Sections covered" list and identify which type of questions I mention for each section.

Sections not on exam. The following sections are *worth rereading as review*:

1.5 A good example of using Taylor's Theorem.
1.6 Ditto.
3.3 Good reading for understanding Newton's method, but otherwise too specific.
3.4 An excellent example of Newton's method, linear interpolation, and machine epsilon.
3.7 Ditto.

We will return to sections 2.5, 2.6, and 2.7 later. Sections 1.4, 1.7, 1.8, 3.10, 3.11, 3.12 are permanently skipped.