Review Guide for In-Class Final Exam

on Tuesday, 12 December 2017, from 10:15am to 12:15pm in Reichardt 202

The Exam is *closed book* and *no calculator*.
You may bring *one letter-paper-sized sheet* of notes.

The Final Exam will cover the following sections of the textbook J. Epperson, *An Introduction to Numerical Methods and Analysis*, 2nd edition:

1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 2.4, 3.1, 3.2, 3.5, 4.1, 4.3,
2.5, 5.1, 5.3, 5.4, 5.6, 6.1, 6.2, 6.4, 7.1, 7.2, 7.3.

A more detailed section and topic list is given below. My goal is to only include topics that have appeared on homework and in lecture, and to only ask questions which can be answered quickly by a prepared student.

The sections listed in the *second* line above, i.e. 2.5–7.3, will be emphasized. Those sections in the first line were covered on the Midterm Exam, so they are de-emphasized. However, ideas from sections 1.1–4.3 will still appear.

Strongly recommended: Get together with other students and work through this Review. Be honest with yourself about what you do and don't know. Talk it through and learn!

Sections and topics. Questions are limited to the following sections and topics:

- 1.1 Be able to state and apply Taylor's theorem with remainder (Theorem 1.1).
- 1.2 Know the definitions of absolute error, relative error, and $O(h^q)$.
- 1.3 Know the definition of machine epsilon.
- 2.1 Be able to write Horner's method as a pseudocode or use it to rewrite a polynomial.
- 2.2 Be able to derive finite difference formulas like (2.1) and (2.4) from Taylor's theorem.
- 2.3 Be able to sketch, write as a pseudocode, and do a couple of steps of Euler's method.
- 2.4 Be able to derive (2.11). Be able to apply Theorem 2.1 (if it is stated for you).
- 3.1 Be able to sketch, write as a pseudocode, and do steps of the bisection method.
- 3.2 Be able to derive, sketch, write as a pseudocode, and do steps of Newton's method. Understand relative advantages/disadvantages of bisection and Newton's methods.
- 3.5 Know definitions: order of convergence p, superlinear and quadratic convergence.
- 4.1 Understand what Theorem 4.1 is saying. Given data $\{(x_j, f(x_j))\}_{j=0}^n$, be able to write down Lagrange's functions $L_i^{(n)}(x)$ and the polynomial $p_n(x)$.
- 4.3 Understand and be able to state the polynomial interpolation error theorem (Theorem 4.3), noting its close analogy to Theorem 1.1. Be able to apply Theorem 4.3. Know the definition of $\|f\|_\infty$.

- 2.5 Be able to derive, sketch, write as a pseudocode, and do steps of the trapezoid rule, both a single step and the composite (uniform-grid) rule. Understand how Theorem 2.1 is used to derive Theorem 2.2 (if these are stated for you).
- 5.1 Know what a Riemann sum is and how to sketch one.
- 5.3 Be able to derive, sketch, write as a pseudocode, and do steps of Simpson's rule. Know the definition of degree of precision. Be able to apply Theorem 5.1 and/or Corollary 5.1 (if they are stated for you).
- 5.4 Be able to derive, sketch, write as a pseudocode, and do steps of the midpoint rule. Be able to apply the formulas on the bottom of page 284 (if they are stated for you).
- 5.6 Understand and be able to describe the goal of Gaussian quadrature. Be able to write a pseudocode which would implement Gaussian quadrature in small n cases (e.g. given a table of nodes and weights).
- 6.1 Understand, and understand the importance of, Theorem 6.1; it says that most ODE IVPs are usable for predictions. Be able to convert a scalar ODE of order n into a first-order system of n equations.
- 6.2 Just as for section 2.3: Be able to sketch, write as a pseudocode, and do a couple of steps of Euler's method.
- 6.4 Be able to sketch, write as a pseudocode, and do a couple of steps of backward Euler, the midpoint method, and the trapezoid rule *in easy cases* if these methods are stated for you. Know what explicit and implicit mean. Be able to sketch, write as a pseudocode, and do a couple of steps of the trapezoid rule predictor-corrector method *in easy cases* if it is stated for you.
- 7.1 Be able to read and use indices for matrix and vector notation in the standard ways, as on page 418. Understand Theorem 7.1, especially the equivalence of 1, 4, and 6. Know what a triangular matrix is.
- 7.2 Be able to write as a pseudocode, and apply to easy examples, Gaussian elimination (Alg. 7.1) and back-substitution (Alg. 7.2). Be able to use partial-pivoting on an easy example.
- 7.3 Be able to count operations in simple linear algebra operations (e.g. dot product $x \cdot y$; matrix-vector product Ax) and in simple pseudocodes (e.g. back-substitution). Understand and be able to use "$O(n^q)$" notation correctly for these algorithms.

Types of questions. The exam will have these types of questions:

(a) state Theorems 1.1 and 4.3 $\leftarrow$ *know these Theorems by name, not number*
(b) state definitions (if mentioned above)
(c) apply theorems if mentioned above and if they are stated for you on the exam
(d) sketch the idea/derivation of a method (if mentioned above)
(e) do calculations which apply a method (if mentioned above)
  - especially: do one or two steps of iterations on easy examples

(f) write pseudocodes of methods (if mentioned above).