Assignment #8

Due Thursday, 9 November at the start of class

Please read sections 5.1–5.4, and 5.6 of the textbook (J. Epperson, *An Intro. to Numerical Methods and Analysis*, 2nd edition).

Section 5.3, pages 280–282:

- **Exercise 3.** The question about trapezoid rule is answered by Exercise 6 in Section 2.5. No need to redo that, but do use the solutions to Assignment # 7 to write down a comparison of results.
- **Exercise 6.** Do parts (a) and (b) only. It is recommended that you write one function, which takes f, a, b, n as inputs, and apply it to each part.
- **Exercise 10.**
- **Exercise 15.** That is, derive Simpson's 3/8 rule.

Section 5.4, pages 285–286:

- **Exercise 3.**
- **Exercise 6.**
- **Exercise 10.**

P10. Suppose $f(x)$ is defined on $\mathbb{R}$, has a continuous derivative, and is also periodic with period $L > 0$. "Periodic with period L" means that $f(x+L) = f(x)$ for any $x \in \mathbb{R}$.

(a) Show that $f'(x) = f'(x+L)$ for any x. (*Hint: First use the definition of* $f'(x+L)$.)

(b) Show that the corrected trapezoid rule $T_n^C(f)$, defined on page 267, is the same as the original trapezoid rule $T_n(f)$ if they are applied on an interval $[a, b]$ where $b - a = L$. This means that the original trapezoid rule is extra-accurate for smooth, periodic functions *if* we are computing an integral over a full period L.

(c) For example, compare these two integrals using a composite trapezoid rule code such as `trap.m` from the solutions to Assignment #7:

$$\int_0^2 e^{\sin(\pi x)}\,dx = 2.53213175550403,$$
$$\int_0^2 e^{-x^2}\,dx = 0.882081390762422.$$

(*I assert the values on the right are exact to at least 14 digits.*) Show the difference in convergence rate, for the composite trapezoid rule applied to each of the above integrals, by plotting the absolute error for $n = 1, 2, 4, 8, 16, 32$. That is, use h on the x-axis, absolute error on the y-axis, and show results for each integral in the same `loglog` plot. Use commands such as `legend`, `xlabel`, `ylabel`, `grid`, and etc. to make it clear.