Assignment #5

Due Thursday, 12 October at the start of class

Please read sections 3.8–3.9 and 4.1–4.3 of the textbook (J. Epperson, *An Intro. to Numerical Methods and Analysis*, 2nd edition). Note sections 3.10–3.13 are skipped.

Section 3.9, pages 134–135:

- **Exercise 1**
- **Exercise 4** To show that it converges for all $x_0 \in [1, 2]$, use parts 1 and 2 of Theorem 3.5.
- **Exercise 5** Do parts (a) and (b) only.

Section 4.1, pages 176–177:

- **Exercise 1**
- **Exercise 3**
- **Exercise 5** Do parts (a) and (b) only.

P4. Write a code `secant.m` with first line

```
function x = secant(f,x0,x1,tol)
```

which implements the secant method in section 3.8.

Though the pseudocode on page 125 can be used, it is better in my opinion to require $|f(x)| <$ `tol` instead of the proposed test $|x_1 - x_0| <$ `tol`. Also, there is no reason to optimize your code (as does the pseudocode). On the other hand, your code will be more reliable if you add a checks for division by zero and for other special cases one might think of. In any case, you will get full credit if your code works correctly on the following two examples. Use `tol` $= 10^{-8}$ in both cases.

(a) Solve $f(x) = 0$ where $f(x) = x^3 - x^2 + 3x - 10$ and $x_0 = 0$ and $x_1 = 1$.

(b) Solve $x = \cos(x)$ where $x_0 = 1$ and $x_1 = 1.5$.

P5. On page 124 of the textbook there is the approximation

$$|\alpha - x_{n+1}| \approx C|\alpha - x_n||\alpha - x_{n-1}|. \tag{1}$$

when n is sufficiently large. On the other hand, the order of convergence of the secant method is p, which means that

$$\frac{|\alpha - x_{n+1}|}{|\alpha - x_n|^p} \approx A \tag{2}$$

when n is sufficiently large, where A is neither 0 or ∞.

One may find the precise order of convergence of the secant method by combining these facts. Specifically, define $z_n = |\alpha - x_n|$ for simplicity. You will assume that the secant method is converging so that the numbers z_n are arbitrarily small. Then turn both (1) and (2) into equalities. (*This step is only justified in the limit, but here you should just do it.*) By combining these equalities, and taking logarithms as needed, one may find p. It will satisfy $1 < p < 2$.

P6. (*This problem asks you to be a bit creative. It is not worth many points, and it will require some time, so do it last. I will be very generous with points if I see reasonable-and-serious-but-failed attempts!*)

Recall that the bisection method is robust because it maintains a bracket around the solution, assuming that one is given initially. On the other hand, the secant method is efficient because it converges superlinearly. Note that both bisection and secant are easier to use than Newton in the sense that they only need the function $f(x)$, but not the derivative $f'(x)$.

Build a code with first line

```
function x = hybrid(f,a,b,tol)
```

which first checks if $[a, b]$ is a bracket and then combines the bisection and secant methods, *in some way that you create*, that maintains a bracket at each step but which converges to the solution, in good cases, as quickly as the secant method. Test it on the cases given in parts (a) and (b) of problem **P4**, but starting with initial brackets.