Worksheet: degree 4 numerical integration

(a) As warm-up, integrate:

$$\int_a^b c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 \, dt =$$

(b) Build a degree-4-integrator code. It will approximately compute the number $\int_a^b f(t)\,dt$. It will do the following steps:

(i) generate 5 points $t_1, \ldots, t_5$ in $[a, b]$, ← *you can decide which 5 points you like!*
(ii) set up the 5×5 Vandermonde matrix for these t_j,
(iii) compute the coefficients in the degree 4 polynomial $p(t)$ so that $p(t_j) = f(t_j)$, and
(iv) return the exact integral $\int_a^b p(t)\,dt$ as the approximation to $\int_a^b f(t)\,dt$.

It will not call any MATLAB black boxes except "`A\b`". For instance, you cannot use `vander`, `polyfit`, or `polyval`. Fill this in:

```
function z = deg4int(f,a,b)
% DEG4INT  Approximates the integral of f(t) on [a,b] using a degree 4
% polynomial interpolant.
% Example: >> deg4int(@(x) sin(x),0,pi)   % exact = 2.0000000
```

(c) For those groups with a computer in hand, type in the above code. Test it on $\int_0^\pi \sin(x)\,dx$. Also compare to the built-in integrator `quad`.