

Worksheet: Fast computation of square roots

(a) Without using a calculator or computer, and without expending much effort, estimate the square root of $x = 4.4234 \times 10^{38}$. (*Hint*: Note that the exponent is *even*!)

(b) Brainstorm with your group: Suppose the positive number x has expression¹

(*)
$$x = a \times 2^b$$

where $1 \leq a < 2$ and b is an integer. *ALSO ASSUME b IS EVEN* for this part. Using form (*), decide how to *quickly* compute a highly-accurate approximation of \sqrt{x} , say with 14 decimal digit accuracy, using only the elementary operations of addition, subtraction, multiplication, and division.

(c) Using the result of (b), fill in this function that computes square roots using only the elementary operations. Decide how to address $x = a \times 2^b$, but with b either even or odd.

```
function z = mysqrt(x)
% MYSQRT Computes the square root z of a positive input x.
if x < 0, error('MYSQRT only works for x >= 0'), end
if x == 0, z = 0; end
[a,b] = ieeeparts(x); % so x = a 2^b as in (*) above
```

¹This is the form the positive number x would have if it were written in IEEE floating point representation,

$$x = \begin{bmatrix} 0 & e_1 & e_2 & \dots & e_{11} & b_1 & b_2 & b_3 & \dots & b_{52} \end{bmatrix} = + (1.b_1b_2b_3 \dots b_{52})_2 2^{(e_1e_2 \dots e_{11})_2 - 1023_{10}}$$