Study Guide for Final Exam

The Final Exam is on Friday, 14 December from 10:15 am–12:15 pm.

This exam is in-class and closed-book. No calculators or computers or phones are allowed. You may bring a 1/2 sheet of letter size paper with any notes you want on it.

This exam covers the whole course, but section 7.2 or later will be emphasized because earlier material appeared on the Midterm.

Problems will be in these categories:

- apply an algorithm/method in a simple concrete case,
  E.g. Use the equally-spaced composite trapezoid rule with $n = 3$ *subintervals to do this integral, and compare to the exact answer.*
- apply an error theorem in a simple concrete case,
  E.g. Apply this stated error theorem for piecewise-linear interpolation [as on Assignment #8] *to estimate the error in approximating this function at these points.*
- state a theorem or definition from memory,
  E.g. Define the Chebyshev points on the interval $[-1, 1]$. *Or: State the polynomial interpolation error theorem.* (I will not ask you to *prove* theorems. The definitions and the two theorems you should memorize are listed below.)
- write a short MATLAB code to state an algorithm,
  E.g. Write Newton's method as a MATLAB *code.* (Write it based either on your memory/understanding of the algorithm, or from a given description.)
- derive an algorithm or explain an idea.
  E.g. Derive the trapezoid rule starting with the idea that it uses two-point interpolation. Or: Why is one of these methods better than another, when applied to this example? (Write in complete sentences.)

How will I create the exam? I'll ask: "Did a question like that appear on homework?"

Sections. From the textbook by Greenbaum & Chartier, see these sections that we covered recently in lecture and homework:

7.2 (ignore 7.2.4 and 7.2.5), 7.3, 7.4 (esp. norms and condition #), 7.6 (ignore 7.6.2),
8.1, 8.2, 8.4, 8.5, 8.6 (ignore 8.6.1),
10.1, 10.2, 10.3, 11.1, 11.2.

Of these, the most important sections introduced new problems, basic algorithms, and the polynomial interpolation error theorem: 7.2, 7.6, 8.4, 8.6, 10.1, 10.2, 10.3, 11.2.

Please lightly re-examine the sections before the Midterm (2.1–2.10, 4.1–4.5, 5.2–5.4, 6.1–6.2, and 7.1) and the online notes *How to put a polynomial through points*.

Definitions. Please recall these definitions from memory.

- *absolute* and *relative error* (chapter 6, p. 124)
- *absolute* and *relative condition number* (section 6.1)
- *norm* for vectors and matrices (section 7.4)
- *condition number* for matrices (section 7.4, p. 160)
- *normal equations* (section 7.6, p. 167, equation (7.14))
- *Chebyshev points on* $[-1, 1]$ (formula (8.15) on p. 192)
- *cubic spline* (subsection 8.6.2; definition fully written out in lecture)
- *natural cubic spline* (ditto)
- *ordinary differential equation initial value problem* (ODEIVP) (eqns. (11.1) on p. 251)

Theorems and Error Formulas. You should understand the statements of the theorems, and be able to apply them in particular cases. I will not ask you about the proofs. For those without "MEMORIZE," I will reproduce the statement on to the exam.

- Taylor's theorem with remainder (Thm 4.2.1) ⟵MEMORIZE
- polynomial interpolation error theorem with remainder (Thm 8.4.1) ⟵MEMORIZE
- piecewise-linear interpolation error estimate (last eqn. on p. 198 in section 8.6)
- error formulas for trapezoid and Simpson's rules (see Table 10.3 on p. 235)

Algorithms. You need to be able to recall these algorithms from memory, or re-derive them as needed.

- bisection method (section 4.1)
- Newton's method (section 4.3)
- Gaussian elimination with partial pivoting (section 7.2)
- forward and substitution to solve triangular linear systems (section 7.2)
- normal equations for least squares problems (section 7.6)
- fitting polynomials to data (section 7.6)
- constructing the interpolating polynomial (online notes and sections 8.1,8.2):
  - Vandermonde matrix method
  - Lagrange's direct formula for the polynomial
- piecewise-linear interpolation (section 8.6)
- cubic spline interpolation (section 8.6)
- Newton-Cotes formulas: trapezoid, Simpson's, midpoint (section 10.1)
- composite versions of above (section 10.2)
- Gauss quadrature (section 10.3)
- Euler method for ODE IVPs (section 11.2)
- midpoint method for ODE IVPs (section 11.2)

Concepts.

- Count operations in simple algorithms. (*E.g. Exercise 6 in Chapter 7.*)
- Why are the Chebyshev points superior for polynomial interpolation? (*Hint*: In the remainder term in Theorem 8.4.1, what is small if you use Chebyshev?)
- What is the basic idea behind all the numerical integration methods we studied? (*Answer*: Replace the integrand $f(x)$ by a polynomial or piecewise-polynomial interpolant $p(x)$, and integrate that.)