Assignment #9

Due Tuesday, 11 December, 2012 at 5:00 pm
in my office box (Chapman 101, DMS office)

In your solutions to all problems, remember to:

- show me your calculations,
- show me the MATLAB code or commands, if used, and
- check that you have answered the question.

These problems are on numerical integration. Refer to Chapter 10 of the textbook.

Exercise 10.2 on page 248.

Exercise 10.4 on page 249.

Exercise 10.7 on page 249.

Problem I. Gaussian quadrature can be applied to do integrals on infinite intervals using only finitely many interpolation (quadrature) points. For example, "Gauss-Hermite quadrature" does integrals of the form

$$\int_{-\infty}^{+\infty} f(x) e^{-x^2}\,dx$$

by a weighted sum of the values of f:

$$\int_{-\infty}^{+\infty} f(x) e^{-x^2}\,dx \approx \sum_{i=1}^{n} w_i f(x_i).$$

Here n is the number of quadrature points, $x_1, \dots, x_n$ are the locations of those points (the "abscissae"), and $w_1, \dots, w_n$ are called "weights".[1] At my website you will find a code which computes the abscissae x_i and the weights w_i:

`http://www.dms.uaf.edu/~bueler/hermquad.m`

(a) Check that the $n = 2$ case is exact for $f(x) = 1, x, x^2, x^3$ using the fact that

$$\int_{-\infty}^{\infty} x^n e^{-x^2}\,dx = \begin{cases} 0, & n \text{ odd}, \\ \sqrt{\pi}, & n = 0, \\ \frac{1}{2}\sqrt{\pi}, & n = 2. \end{cases}$$

(I wrote a very short MATLAB code to do this job.)

(b) Apply the $n = 6$ case to do

$$\int_{-\infty}^{\infty} \cos(x) e^{-x^2}\,dx.$$

[1] Sorry about the old-fashioned language. Learning the old language is merely culture, not truth.

Compare to the value computed by MATLAB's built-in black box but using a truncated interval of integration:

```
>> quad(@(x) cos(x) .* exp(-x.^2),-20,20)
```

Let's assume the answer from `quad` is exact. Noting that $f(x) = \cos(x)$ is *not* a polynomial, explain how the answer from $n = 6$ point Gauss-Hermite quadrature can be so good given how few quadrature points (abscissae) are used to evaluate $f(x)$, and the fact that the abscissae are quite close to the origin while the limits of integration are $-\infty$ and ∞ are far away! (*Hint*: Have you drawn a picture of this integral?)

Problem II. Write your own routine using `polyfit` to do N-point Clenshaw-Curtis quadrature for integrals on the interval $[a, b] = [-1, 1]$. (Do not use `chebfun` even though it is fun!) I suggest you use the line

```
x = cos(pi*(0:N)/N);
```

to generate the interpolation (quadrature) points. Apply your method with $N = 4, 6, 10$ to compute this integral:

$$\int_{-1}^{1} x \sin(x)\, dx.$$

Compare to the exact value; compute that exact value *by hand*!

These problems are on numerical solutions of ordinary differential equations. Refer to Chapter 11 of the textbook.

Exercise 11.1 on page 295. Do only parts (a) and (b).

Exercise 11.3 on page 295.

Exercise 11.4 on page 295.

Exercise 11.6 on page 295.