Assignment #5

Due Wednesday 24 October, 2012 at the start of class, when I will hand out solutions immediately. Thus: *late homework will not be accepted.*

In your solutions to all problems, remember to:

- show me your calculations,
- show me the MATLAB code or commands, if used, and
- check that you have answered the question.

Problem I. **(a)** Consider the system

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 5 \\ 4x_1 - 3x_2 + 2x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 3 \end{aligned}$$

As is done with the example on pages 134–135 (i.e. the first example in section 7.2), perform Gaussian elimination *by hand* to transform it into an upper triangular system. (Do not swap rows.) Then do back substitution *by hand*.

Of course, the system has abstract form $A\mathbf{x} = \mathbf{b}$. Enter A and $\mathbf{b}$ into MATLAB and confirm your by-hand solution by `x = A \ b`.

(b) Let

$$L_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{2}{9} & 1 \end{bmatrix}.$$

Let M be the augmented matrix $M = \left[A \middle| b\right]$. Confirm *by hand* that

$$L_1 M \qquad \text{and} \qquad L_2 L_1 M$$

correspond to stages of the computation you did by hand in part **(a)**. In particular, $L_2 L_1 M$ is the upper triangular system you solved in **(a)**.

(c) *All of the computations in this part can and should use* MATLAB*. You may us the* `inv` *command*: Compute $U = L_2 L_1 A$. Check that

$$(L_1)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \qquad (L_2)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{2}{9} & 1 \end{bmatrix}.$$

Confirm that $(L_2 L_1)^{-1} = (L_1)^{-1}(L_2)^{-1}$. Let $L = (L_2 L_1)^{-1}$. Confirm that $LU = A$.

(d) At the bottom of page 135 it says "…to solve $A\mathbf{x} = LU\mathbf{x} = \mathbf{b}$, one first solves the lower triangular system $L\mathbf{y} = \mathbf{b}$ (to obtain $\mathbf{y} = U\mathbf{x}$) and then the upper triangular system $U\mathbf{x} = \mathbf{y}$." Confirm by doing it in MATLAB that you get the same answer this way as you did in part **(a)**.

Problem II. **(a)** Open a new m-file called `firstge.m`. Type in the code at the top of page 137; you don't need to type in the comments, but be careful with the loop indices and other details. Note that, for the code to run, the variables `n, A, b` must all be already defined and of the right size in order for this to run properly.

(b) Consider the 4×4 linear system $A\mathbf{x} = \mathbf{b}$:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 7 \\ 2x_1 + x_2 \qquad\qquad - x_4 &= -1 \\ x_1 \qquad\qquad\qquad + x_4 &= 4 \\ 2x_2 - 2x_3 \qquad\quad &= -8 \end{aligned}$$

Type it in and run `firstge.m` on it. Show the resulting A and $\mathbf{b}$; these are different from the ones you typed in. By computing `x = A \ b`, solve this new system. Confirm by `x = A \ b` on the *original* A and $\mathbf{b}$ that you are getting the same solution. Also confirm your solution by hand based on the original system. (*Note*: Confirming a solution *does not* require doing Gaussian elimination by hand! You will not get full credit if you do that.)

The remaining problems are from Chapter 7 of the textbook *Numerical Methods: Design, Analysis, and Computer Implementation of Algorithms* by Greenbaum and Chartier:

Exercise 2 on page 175.

Exercise 3 on pages 175–176.

Exercise 4 on page 176.

Exercise 6 on page 176.

Exercise 8 on page 176.