Study Guide for Final Exam

The Final Exam is in-class at 10:15 am–12:15 pm, 16 December, 2011.

The exam is closed-book and no calculators are allowed.

Problems will be in these categories:

- apply an algorithm/method in a simple concrete case,
- apply an error theorem to a particular case,
  Except for "MEMORIZE" below, I will state the error theorem as part of the problem.
- state a theorem or definition,
  Applies to definitions and those theorems with "MEMORIZE". I will not ask you to prove *theorems.*
- write a short pseudocode or a MATLAB/OCTAVE code to state an algorithm,
- explain/show in words, and
  e.g. Why is one method is better than another, when applied to a particular example? Write in complete sentences.
- derive an algorithm.

Sections. From the textbook J. Epperson, *An Introduction to Numerical Methods and Analysis*, see these sections that we covered in lecture and homework:

review:	1.1, 1.2,
general numerical ideas:	1.3, 2.1,
root-finding:	3.1, 3.2, 3.3, 3.5, 3.6, 3.8,
interpolation:	2.4, *How to put a polynomial ...* online notes, 4.1, 4.2, 4.3,
integration:	2.5, 5.1, 5.2, 5.3, 5.4, my lecture on Romberg integration,
linear systems:	2.6, 7.1, 7.2, 7.3,
differential equations:	2.3, 6.1, 6.2.

Definitions. You need to be able to recall these definitions from memory.

- absolute and relative error (p. 15)
- order of convergence for sequences (p. 110)
- degree of precision of a quadrature rule (p. 265)
- a matrix A is nonsingular (or A is singular) (p. 408)

Algorithms ("how to" aspects). Generally you need to be able to recall these algorithms from memory, or rederive them as needed.

- Horner's rule (Alg. 2.1 on p. 40)
- linear interpolation (p. 58)
- bisection (Alg. 3.1 on p. 89 and/or Alg. 3.2 on p. 91)
- Newton's method (equation (3.7) on p. 96)
- secant method (Alg. 3.3 on p. 123)

Algorithms, cont.

- methods to construct polynomial interpolant; all are in "*How to put a polynomial ...*" notes:
  - Lagrange polynomials (Theorem 4.1; p. 162)
  - Vandermonde method
  - Newton form of polynomial (section 4.2; p. 168)
- trapezoid rule (p. 64 and p. 256)
- Simpson's rule (pp. 262–263)
- midpoint rule (p. 273)
- Romberg integration (as done in lecture; ideas, not formulas)
- (naive) Gaussian elimination (Alg. 7.1 on p. 411)
- backward substitution (Alg. 7.2 on p. 412)
- Gaussian elim. with partial pivoting (Alg. 7.3 and Alg. 7.4 on pp. 413–414)
- Euler method for solving ODE IVP (bottom line of p. 329)

Theorems and error formulas (mostly "how good" aspects). I care whether you understand the statements of the theorems, and whether you can apply them in particular cases.

- Taylor's Theorem with Remainder (pp. 2–3) ←<u>MEMORIZE</u>
- Intermediate Value Theorem (p. 10)
- Mean Value Theorem (p. 9)
- linear interpolation error theorem (p. 60)
- bisection convergence and error theorem (p. 89)
- Newton error formula theorem (pp. 108–109)
- Theorem 3.3, Newton convergence theorem (p. 114)
- Theorem 4.1, existence and uniqueness thm. for poly. interpolation (p. 162)
- Theorem 4.3, polynomial interpolation error theorem (p. 176) ←<u>MEMORIZE</u>
- trapezoid rule error theorems:
  - single interval: p. 66 and p. 256
  - composite (uniform): p. 66
- Simpson's rule error theorems:
  - single interval: Theorem 5.1, p. 266
  - composite (uniform): Corollary 5.1, p. 267
- midpoint rule error theorems:
  - single interval: p. 274
  - composite (uniform): p. 274
- fundamental theorem of linear algebra, Theorem 7.1 (p. 408)

Other concepts.

- Floating point arithmetic (as described on pp. 21–23)
- the work in solving a nonsingular linear system of n equations is dominated by the Gaussian elimination stage; the number of arithmetic operations is $\frac{2}{3}n^3 + O(n^2)$, where $O(n^2)$ is "small change" (p. 418)