Assignment #7

Due Wednesday 23 November, 2011 at the start of class

Read subsections 5.1, 5.2, 5.3, and 5.4 of the text.[1] Then do the following exercises:

Page 255, Exercise 1. (Note that "$I_n(f) = \sum_{i=0}^{n} w_i f(x_i)$" should be thought of as one of the numerical integration (quadrature) rules, like trapezoid or Simpson's. This problems asks you to show that numerical integration rules of this form are linear.)

Page 255, Exercise 2.

Page 270, Exercise 2.

Page 271, Exercise 10.

Page 275, Exercise 6.

Page 276, Exercise 10.

P7. Use the $n = 4$ cases of the

- trapezoid rule $T_n(f)$,
- the corrected trapezoid rule $T_n^C(f)$,
- Simpson's rule $S_n(f)$,
- and the midpoint rule $M_n(f)$

on the integral

$$\int_0^1 x(1-x^2)\,dx.$$

Also compute the exact integral, and then the actual error from each rule. Are the results as expected? (*In particular, are any of the approximations actually exact, and if so, why? Are any of the approximations known in advance to be below or above the exact value?*)

P8. **(a)** How small does h need to be to get accuracy of 10^{-8} for the integral

$$\int_0^5 \sin(x)\,dx$$

if Simpson's rule is used? How many points must be used?; remember that this number must be even for Simpson's rule. Answer the same questions for trapezoid rule.

(b) Redo part **(a)** for the integral

$$\int_0^5 \sin(20x)\,dx.$$

Then explain why this integral is harder for both rules.

[1] That is, J. Epperson, *An Introduction to Numerical Methods and Analysis*, rev. ed., 2007.

P9. Write a reasonably brief program which uses Simpson's rule to build a table of values of the Fresnel integrals

$$C(x) = \int_0^x \cos(\pi t^2/2)\, dt$$

at the 21 locations $x = 0, 0.1, 0.2, 0.3, \dots, 2.0$. Describe how would you evaluate the accuracy of the resulting table.

P10. Derive trapezoid rule the way that Simpson's rule is derived in section 5.3, on pages 261–262. Specifically, let $p_1(x)$ be the linear polynomial that interpolates $f(x)$ at $x_0 = a$ and $x_1 = b$. Define

$$T_1(f) = I(p_1) = \int_0^1 L_0(x) f(a) + L_1(x) f(b)\, dx.$$

State $L_0(x)$ and $L_1(x)$ in this case. Find A, B, as in the Simpson's rule derivation, so that

$$T_1(f) = A f(a) + B f(b).$$

(*You will integrate straight lines to get A and B.*)