Assignment #5

Due Wed 26 October, 2011 at the start of class

Read subsections 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, and 3.9 of the text.[1] Then do the following exercises:

Page 102, Exercise 3ab. (Make sure to compare to bisection.)

Page 112, Exercise 3.

Pages 112–113, Exercise 5.

Page 113, Exercise 6.

Page 113, Exercise 7.

Page 116, Exercise 1.

Page 116, Exercise 2.

Page 119, Exercise 1.

Page 123, Exercise 1.

P3. Write a MATLAB/OCTAVE function

```
function z = nthroot(x,n)
```

which computes $z = x^{1/n}$ by solving $f(z) = z^n - x = 0$ for z. Your function will use the given x and n as constants, and will use Newton's method. Have the program stop immediately if $x < 0$ or $n < 1$; use the `error` function in MATLAB/OCTAVE to stop. Thus the rest of your code may assume $n \geq 1$ and $x \geq 0$. Come up with a "fair" scheme for getting an initial guess, where "fair" means that inside your code only operations $+, -, \times, \div$ are used for the computation, except that powers a^n and a^{n-1} may appear inside your code. (Note that if n is a small positive integer, e.g. $n = 1, 2, 3, 4, 5, \dots$, then these powers *could* be computed by multiplication, e.g. $a^4 = a \times a \times a \times a$. So go ahead and use nth and $(n-1)$st powers in your code when needed.)

P4. Will your buried pipe freeze when the air temperature drops? (*This is a rewrite of problem 6 on page 103, which is not clear. This problem has a different answer.*)

Assume the initial air and ground temperature is $T_0 = 10°$ C. Assume the air temperature drops suddenly to $T = -30°$ C. A simple model, which appears in Math 421, for the temperature $u = u(x,t)$ at depth x and time t after the shift, is

$$u(x,t) = T + (T_0 - T)\,\mathrm{erf}\left(\frac{x}{2\sqrt{at}}\right).$$

[1] That is, J. Epperson, *An Introduction to Numerical Methods and Analysis*, rev. ed., 2007.

Suppose $a = 1.25 \times 10^{-6}$ ft^2/sec, which is the conductivity (actually "thermal diffusivity") of your soil. Find the depth x so that the temperature at that depth does not reach 0° C after a time t of 30 days after the shift of 40° C. That is, how deep should you bury the pipe if the standard is to last 30 days after such a temperature shift?

Redo the problem assuming $T = -50^\circ$ C, that is, using a 60° C drop. Redo the problem assuming t is 90 days, but the shift is the original one, so $T = -30^\circ$ C.

Use Newton's method and explain how you got a first guess. Note `erf` is a command in MATLAB/OCTAVE. If you need the derivative of the function $\mathrm{erf}(x)$, use the definition of that function on page 30, and the fundamental theorem of calculus.