SAMPLE Midterm Exam #1

1. Show that $y = \sqrt{x - \frac{1}{2} + \frac{1}{2}e^{-2x}}$ is a solution of the initial value problem

$$\frac{dy}{dx} = \frac{x}{y} - y, \qquad y(0) = 0.$$

2. Solve for $\theta(t)$:

$$\frac{d\theta}{dt} - 6\theta = t.$$

3. Check that the following equation is exact, and then solve the initial value problem:

$$(2x + y)\,dx + (x - 2y)\,dy = 0, \qquad y(1) = 2.$$

4. Solve the following equation explicitly for $y = y(x)$.

$$\frac{dy}{dx} = 4 + y^2.$$

5. Use Euler's method with steps of size $h = 1.0$ to approximate the solution of the initial value problem

$$y' = x - y^2, \qquad y(1) = 0$$

at $x = 2$ and $x = 3$.

6. Solve:

$$\frac{dy}{dx} = -3 + \frac{y}{x}.$$

7. (a). We count burbot (which are really ugly fish that live in the Tanana River among other places) and discover that in 1990 there were 10,000 in the river and that in 1995 there were 12,000. Using the Malthusian model estimate the population in 2000.

(b). Write down the differential equation for part a) (even if you have already!). Then write down a new differential equation for the model: We assume that the population of burbot grows at a rate proportional to the current population minus a constant amount of fishing, which we assume to be 1000 per year.

(c). Extra Credit. Solve the model in (b). Then explain how to use the data in (a) to find k and any other unknown constants, and how to estimate the population in 2000.

8. The air in a small room 10 ft by 10 ft by 10 ft is 3% carbon monoxide. Starting at $t = 0$, fresh air containing no carbon monoxide is blown into the room at a rate of 100 ft^3/min. The air in the room is kept well-mixed, and air flows out of the room through a vent at the same rate. Determine the time in minutes when the room will have only 0.01% carbon monoxide; your expression for the time may have an elementary function in it.