MATH 302 Differential Equations (Bueler) 2 February 2009

Selected Solutions to Assignment #1

These problems were graded at 3 points each for a total of 27 points.

1.1 #4. PDE, second order, dependent variable is u, independent are x and y

1.1 #14.

$$\frac{dx}{dt} = kx^4$$

1.2 #4. Here $x' = -2\sin t - 3\cos t$ and $x'' = -2\cos t + 3\sin t$ so

$$x'' + x = (-2\cos t + 3\sin t) + (2\cos t - 3\sin t) = 0.$$

Yes, $x(t)$ is a solution.

1.2 #10. Using implicit differentiation, where $y = y(x)$,

$$\frac{dy}{dx} - \frac{1}{y}\frac{dy}{dx} = 2x,$$

$$\frac{dy}{dx} = \frac{2x}{1-\frac{1}{y}} = \frac{2xy}{y-1}.$$

The relation *does* give an implicit solution.

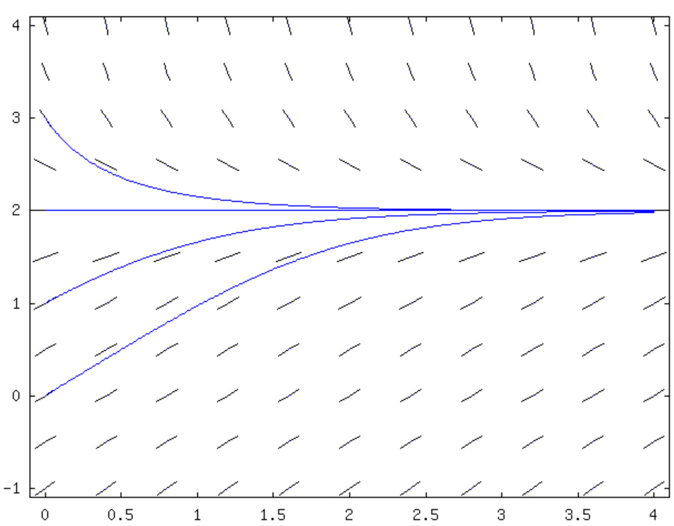
1.2 #24. In standard form,

$$\frac{dy}{dt} = ty + \sin^2 t$$

so $f(t,y) = ty + \sin^2 t$. f is continuous everywhere as a function of t and y, and also $\partial f/\partial y = t$ is continuous everywhere. Thus there is a unique solution to the initial value problem by Theorem 1. (Note we did not even need to look at the initial value to determine if it was a "good" point with respect to the continuity of f or $\partial f/\partial y$. All points in the t, y plane are "good".)

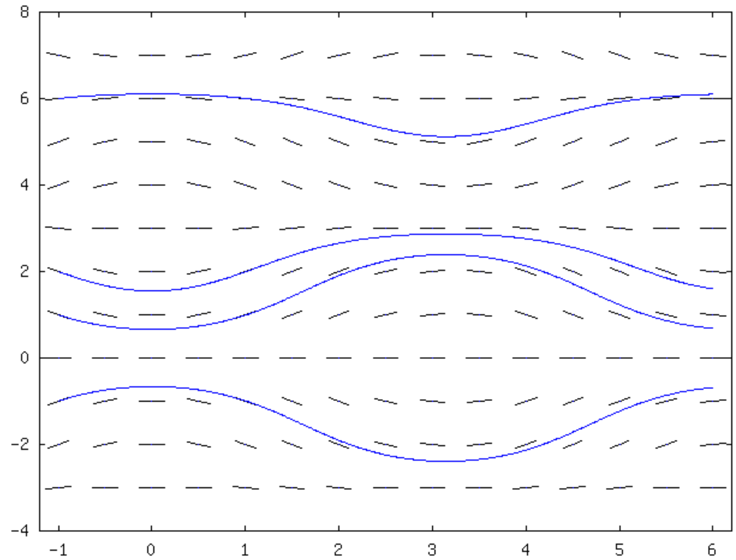
1.3 #3. *Solution in back.*

1.3 #4.



The terminal velocity in this case is $v = 2$. It is found by solving $0 = dv/dt = 1 - v^3/8$.

1.3 #10c.



1.3 #16.