A Solution, from Assignment #11

Recall Assignment #11 is NOT DUE!

8.2 #8. **(d)** We have the series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{2!} - \cdots = \sum_{j=0}^{\infty} (-1)^j \frac{x^{2j}}{(2j)!}$$

It is easiest to apply the original ratio test with $c_j = (-1)^j \frac{x^{2j}}{(2j)!}$. (*Or apply the result of problem 7.*) That is, convergence occurs for all x for which $\lim_{j\to\infty} |c_{j+1}/c_j| < 1$. But

$$\lim_{j\to\infty} \left|\frac{c_{j+1}}{c_j}\right| = \lim_{j\to\infty} \frac{(2j)!|x|^{2(j+1)}}{(2(j+1))!|x|^{2j}} = \lim_{j\to\infty} \frac{|x|^2}{(2j+2)(2j+1)} = 0$$

regardless of x. That is, the power series always converges and the convergence set is $(-\infty, \infty)$. (*If you had computed L by the method in problem 7 then $L = 0$ so $\rho = \infty$.*)

(e) Similar to part **(d)**.